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Identification of the boundary heat transfer coefficient from interior measurement of temperature field

Liyan Wang[1] Binbin Yang Xiaoxuan Yu Cheng Zeng
Department of Mathematics, Southeast University
Nanjing, 210096, P.R.China

Abstract

Consider the heat conduction process for a homogeneous solid rod with one endpoint contacted with some liquid media. The aim is to identify the boundary heat transfer coefficient from the measured temperature field, which is essentially nonlinear and ill-posed. Based on the 1-dimensional heat equation model with Robin boundary condition, we prove the recognizability of the time-dependent Robin coefficient from the temperature field specified at only one interior point. To give a stable reconstruction for noisy inversion input data, we propose a regularizing scheme by minimizing a cost functional including penalty term with rigorous mathematical analysis. A steepest iterative scheme is established to solve this non-quadratic optimizing problem.

Keywords: Inverse problems, heat equation, boundary impedance, uniqueness, optimization, iteration.

1. Introduction

Consider the heat conduction process in a homogeneous solid rod with two endpoints $x = 0, \pi$. For given initial temperature distribution, if one of the endpoint $x = \pi$ of the rod contacts with liquid media such as water, then the heat flux at this endpoint is proportional to the difference of temperatures between solid and liquid (Ch.7, [1]). The temperature field $u(x,t)$ in the rod can then be modeled by

$$\begin{cases} u_t - a^2 u_{xx} = 0, & x \in (0,\pi), t > 0 \\ u(0,t) = 0,\ u_x(\pi,t) + \sigma(t)u(\pi,t) = 0, & t > 0 \\ u(x,0) = f(x), & x \in [0,\pi], \end{cases} \tag{1.1}$$

where the impedance coefficient $\sigma(t) > 0$ represents the heat exchange ability of the rod at $x = \pi$. When $(\sigma(t), f(x))$ is known, the system (1.1) is completely determined, i.e., there exists a unique solution $u(x,t)$ in appropriate function space, see Theorem 7.2.1 in [1].

(1.1) can be applied to simulate the temperature distribution for a metal rod with one of its endpoints contacting with some corrosive liquid such as sea water. Physically, the heat transfer coefficient $\sigma(t)$ is closely related to the property change of the rod endpoint due to the corrosion. Considering that the endpoint $x = \pi$ may not be touchable in many engineering configurations, then an inverse problem arising in the nondestructive detection

[1]Corresponding author: Dr. L.Y.Wang, email: wangliyan@seu.edu.cn.