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The distribution of zeros of oscillatory solutions for second order nonlinear neutral delay differential equations *

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Abstract: In this paper, the distance between adjacent zeros of oscillatory solutions for second order nonlinear neutral delay differential equations

$$(a(t)z'(t))' + q(t)f(x(t - \sigma)) = 0, t \geq t_0,$$

where $z(t) = x(t) + p(t)x(t - \tau)$ is investigated. By means of inequality techniques, specific function sequences and nonincreasing solutions for corresponding first order differential inequality, some new estimates for the distribution of zeros of oscillatory solutions have been presented, which have extended and improved some known results.

Keywords: oscillation; neutral differential equation; distribution of zero.

Mathematics Subject Classification 2010: 34C10, 34K40, 35B05.

1 Introduction

Due to the wide applications in fields of science and engineering, the oscillations of neutral delay differential equations have attracted more and more interest of mathematicians increasingly and have been studied extensively during the past few decades. Here we refer the reader to monographs [1, 2] and papers [3–6].

All above references are concerned with sufficient or necessary and sufficient conditions for oscillations of differential equations, and we will naturally ask some questions: Are there any bounds for the distance between adjacent zeros of oscillatory solutions when equations are oscillatory? And how do we estimate these bounds? As an important branch of oscillation theory, the distribution of zeros of oscillatory solutions for different classes of differential equations which originated in 1950s [7] has aroused great interest of scholars since it gives a unique perspective in the study of properties of solutions. However, because of the difficulties in techniques, the related researches are relatively scarce. For some work in this field, known results can be found in [8–11] and the references cited therein.

In 1997, Zhou and Wang [8] investigate the distribution of zeros of solutions for first order neutral differential equations

$$(x(t) + P(t)x(t - \tau))' + Q(t)x(t - \sigma) = 0, t \geq t_0.$$

An estimate is established for the distance between adjacent zeros of the solutions of such equations under less restrictive hypotheses on the variable coefficients. The results obtained improve and extend some known results in the literature.

In 2014, Saker et al. [10] study the gaps between zeros of nontrivial solutions of fourth order differential equations

$$(r(x)y'''(x))' - (p(x)y'(x))' + q(x)y(x) = 0, x \in I$$

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