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Existence of solution for impulsive differential differential equations with indefinite linear part

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Abstract: In this paper, we consider the existence of solutions for second order impulsive differential equations with Dirichlet boundary conditions and indefinite linear part. An existence result is obtained by using Saddle point theorem, which extends and complements some existing results.

Keywords: Impulsive differential equations; indefinite linear part; Dirichlet boundary conditions; Saddle point theorem.

1. Introduction

In recent years, many researchers have extensively studied the theory and applications of impulsive differential equations, see [1-5] for details. Recently, variational methods have been widely used to study the existence of solutions for impulsive problems (see, [6-21] for details).

Consider the second order impulsive differential equations

$$\begin{cases} -u''(t) = f(t,u),\ t \in [0,1]\backslash\{t_1,t_2,\cdots,t_m\}, \\ u(0) = u(1) = 0,\ u(t_j^+) = u(t_j^-),\ j = 1,2,\cdots,m, \\ u'(t_j^+) = u'(t_j^-) - \bar{I}_j(u(t_j)),\ j = 1,2,\cdots,m, \end{cases} \tag{1.1}$$

where f is a Carathéodory function on $(0,1)\times R$ and will be defined in the following part, $u(t_j^{\pm}) = \lim_{t\to t_j^{\pm}} u(t), 0 = t_0 < t_1 < t_2 < \cdots < t_m < t_{m+1} = 1$, $u'(t_j^+)$ and $u'(t_j^-)$ denote the right and left limits of $u'(t)$ at $t = t_j$ respectively, $\bar{I}_j \in C(R,R), j = 1,2,\cdots,m$. To the author's knowledge, there are few papers considering the existence of solutions for second order impulsive differential equations with indefinite linear part. Motivated by [21,22], we consider problem (1.1) under more relaxed assumptions on $f(t,u)$ than [21]. Our results generalize some existing results in [21].

The rest of the paper is organized as follows. In Section 2, we shall state some lemmas. Criteria for the existence of solution for Eq.(1.1) is established in Section 3.

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