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Leader-following exponential consensus of general linear multi-agent systems via event-triggered control with combinational measurements

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ABSTRACT

In this paper, the leader-following exponential consensus problem of general linear multi-agent systems via event-triggered control is considered. By using the combinational measurements, two classes of event triggers are designed, one depends on continuous communications between the agents, the other avoids continuous communications. For such two classes of event triggers, the exponential consensus as well as the convergence rates of the controlled multi-agent systems are studied, respectively, by employing the M-matrix theory, algebraic graph theory and the Lyapunov method.

1. Introduction

In this paper, we consider the following multi-agent systems including N agents described by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, 2, \dots, N, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^p$ denote the state and control input of agent i, respectively. $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{p\times n}$ are constant matrices. Let $s(t)$ be the leader, labeled by agent 0, whose dynamical behavior is described as follows

$$\dot{s}(t) = As(t). \tag{2}$$

In multi-agent system (1), the control input $u_i(t)$ are event triggered, that is, the control updates are determined by certain events that are triggered depending on the agents' behaviors. The event-triggered controllers have been adopted for control engineering applications, such as the control of memristive systems [1], the control of wireless sensor/actuator networks [2], and the networked control systems [3,4]. In [5–9], the authors have considered the event-triggered control for multi-agent systems. Our work mainly builds on the works in [9,8]. In [9], the authors studied the event-triggered control of integrator multi-agent systems with combinational measurements, in which the graph of communication topology is undirected. In [8], the authors studied the consensus of general linear multi-agent systems using error-based event triggers. Compared with the dynamics of integrator multi-agent systems, the dynamics of general linear multi-agent systems are

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much more complicated: the dynamics of integrator multi-agent systems only depends on the coupling of the agents, the dynamics of general linear multi-agent systems depends not only on the coupling of the agents, but also the self-dynamics governing the evolution of each isolated agent. This makes the consensus of general linear multi-agent systems technically more challenging than the case for integrator agents. Compared with the error-based event-triggers, the event-trigger with combinational measurements avoids the decoupling of the actual states of the agents.

Motivated by the above discussions, we consider the leader-following exponential consensus of general linear multi-agent systems via event-triggered control with combinational measurements. Our work on event-triggered control for leader-following consensus is an extension of the works in [10,11], in which the event-triggered control is not utilized for pinning synchronization of complex networks. The primary contributions in this paper are stated as follows. First, an event-triggered control approach using combinational measurements is applied to the leader-following consensus of general linear multi-agent systems. Second, two classes of event triggers for leader-following consensus are designed. The exponential consensus as well as the convergence rates are considered. Last, for such two classes of event triggers, we prove that the Zeno behavior can be excluded, respectively. Compared with the works in [9], the model considered in this paper is more complicated, which contains both the coupling of the agents and the self-dynamics governing the evolution of each isolated agent. The communication topology is assumed to be directed, which is also an extension of the work in [9]. Compared with the work in [8], the combinational measurement is introduced to construct the event triggers in this paper, different analysis techniques are employed.

Notations. Let $\|x\|$ and $\|A\|$ be the Euclidean norm of a vector x and a matrix A, respectively. Denote I_n be the $n \times n$ identity matrix. Denote $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ be the smallest and largest eigenvalue of symmetric matrix A. For a symmetric matrix A, $A > 0$ means A is positive definite. diag$\{\ldots\}$ stands for a block-diagonal matrix. The superscript "T" represents the vector and matrix transpose. Let $\otimes$ be the Kronecker product.

2. Mathematical preliminaries

2.1. Algebraic graph theory

Let $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ be a weighted digraph of order N with the set of nodes $\mathscr{V} = \{1, 2, \ldots, N\}$, set of directed edges $\mathscr{E} = \mathscr{V} \times \mathscr{V}$, and a weighted adjacency matrix $\mathscr{A} = (a_{ij})_{N\times N}$. An edge in network $\mathscr{G}$ is denoted by (i, j) where i and j are called the terminal and initial nodes, respectively. The adjacency elements associated with the edges of the graph are positive and the others are zero, i.e., $a_{ij} > 0$ if and only if $(j, i) \in \mathscr{E}$, and $a_{ij} = 0$ otherwise. Let $\mathcal{D} = diag\{d_1, d_2, \ldots, d_N\}$, where $d_i = \sum_{j=1,j\neq i}^{N} a_{ij}$. The Laplacian of the weighted digraph $\mathscr{G}$ is defined as $L = \mathcal{D} - \mathscr{A}$.

2.2. Basic definitions, lemmas and assumptions

Definition 1. Leader-following consensus is said to be reached exponentially, if there exist positive constants $\kappa > 0, \gamma > 0$ and $T > 0$ such that

$$\|x_i(t) - s(t)\| \leq \kappa e^{-\gamma t}, \quad i = 1, 2, \ldots, N,$$

for all $t > T$. γ is called the convergence rate.

In the following, we will study the leader-following event-triggered consensus problem of (1). We design the event triggered controller as

$$u_i(t) = K \sum_{j=1}^{N} a_{ij} \left(x_j(t_k^i) - x_i(t_k^i)\right) + K\hat{d}_i \left(s(t_k^i) - x_i(t_k^i)\right), \quad t \in [t_k^i, t_{k+1}^i), \tag{3}$$

where K is the control gain matrix to be determined later. $\hat{d}_i$ is the pinning gain of agent i, satisfies $\hat{d}_i > 0$ if agent i is pinned; $\hat{d}_i = 0$ otherwise. By defining the measurement error of each agent i as $\varepsilon_i(t) = x_i(t) - s(t)$ and introducing the Laplacian Matrix $L = (l_{ij})_{N\times N}$, the multi-agent system (1) controlled by (3) can be rewritten as

$$\dot{\varepsilon}_i(t) = A\varepsilon_i(t) - BK \sum_{j=1}^{N} l_{ij}\varepsilon_j(t_k^i) - BK\hat{d}_i\varepsilon_i(t_k^i), \quad i = 1, 2, \ldots, N. \tag{4}$$

Let $D = diag\{\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_N\}$ be the leader adjacency matrix of the union graph $\bar{\mathscr{G}} = \mathscr{G} \bigcup \{0\}$. For $\bar{\mathscr{G}}$, we say 0 is globally reachable, if there exists a path from every agent i in $\mathscr{G}$ to node 0. Let $\tilde{L} = L + D$ and $\tilde{L} = (\tilde{l}_{ij})_{N\times N}$, then we have the following lemma.

Lemma 1 ([12]). *The eigenvalues of the matrix $\tilde{L}$ have positive real parts if and only if node 0 is globally reachable in graph $\bar{\mathscr{G}}$.*