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Tail-equivalent linearization method for nonlinear random vibration

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Abstract

A new, non-parametric linearization method for nonlinear random vibration analysis is developed. The method employs a discrete representation of the stochastic excitation and concepts from the first-order reliability method, FORM. For a specified response threshold of the nonlinear system, the equivalent linear system is defined by matching the “design points” of the linear and nonlinear responses in the space of the standard normal random variables obtained from the discretization of the excitation. Due to this definition, the tail probability of the linear system is equal to the first-order approximation of the tail probability of the nonlinear system, this property motivating the name Tail-Equivalent Linearization Method (TELM). It is shown that the equivalent linear system is uniquely determined in terms of its impulse response function in a non-parametric form from the knowledge of the design point. The paper examines the influences of various parameters on the tail-equivalent linear system, presents an algorithm for finding the needed sequence of design points, and describes methods for determining various statistics of the nonlinear response, such as the probability distribution, the mean level-crossing rate and the first-passage probability. Applications to single- and multi-degree-of-freedom, non-degrading hysteretic systems illustrate various features of the method, and comparisons with results obtained by Monte Carlo simulations and by the conventional equivalent linearization method (ELM) demonstrate the superior accuracy of TELM over ELM, particularly for high response thresholds.

Keywords: Discretization; Earthquake engineering; First-order approximation; First-passage probability; Hysteresis; Linearization; Nonlinear problems; Nonparametric methods; Random vibrations; Tail probability

1. Introduction

Nonlinear random vibration analysis aims at determining the response statistics of a nonlinear system, when it is subjected to a stochastic excitation. It is useful in predicting the response of structures such as buildings, bridges, or offshore platforms under wind, earthquake, or wave loading. In assessing the safety of a structure, it is important to incorporate the nonlinearity, because failure usually occurs in the nonlinear range of structural behavior.

The topic of nonlinear random vibration has been the focus of much research and development in the past several decades. Methods developed include the Fokker–Planck equation, stochastic averaging, moment closure, perturbation, and equivalent linearization. Recent accounts of these methods can be found in the texts by Roberts and Spanos [20], Soong and Grigoriu [22], Lin and Cai [17], Lutes Sarkani [19]. Among these, the equivalent linearization method has gained wide popularity because of its versatility in application to general, multi-degree-of-freedom nonlinear systems. The other methods, though possibly more accurate, are largely restricted to specialized systems or forms of the excitation, and are difficult to apply in practice. The Monte Carlo simulation method [21] is without restriction, but is computationally demanding.

In the equivalent linearization method (ELM), the nonlinear system of interest is replaced by an equivalent linear system, the parameters of which are determined by minimizing a measure of the discrepancy between the responses of the nonlinear and linear systems [5]. The measure of discrepancy most often used is the mean-square error between the two responses [1,27], although an energy-based measure has also been considered [9]. The solution requires an iterative scheme, since the parameters of the linear system are functions of the second-moments of its response. Furthermore, the method requires an assumption

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regarding the probability distribution of the nonlinear response and most often the Gaussian distribution is selected. As a result, while the method can be quite accurate in estimating the mean-square response, the probability distribution can be far from correct, particularly in the tail region. It follows that estimates of such response statistics as crossing rates and first-passage probability, which are of particular interest in reliability analysis, can be grossly inaccurate at high thresholds. To address this problem, an alternative linearization method was proposed by Casciati et al. [4] by equating the mean level crossing rates of the nonlinear and equivalent linear systems. However, this approach requires knowledge of the joint probability distribution of the response and its derivative, which can be extremely difficult to obtain for general nonlinear systems, particularly those having multiple degrees of freedom.

The method proposed in this paper is also an equivalent linearization method. However, instead of defining the linear system by minimizing the mean-square error in the response, it is defined by matching the tail probability of the linear response to a first-order approximation of the tail probability of the nonlinear response. For this reason, the name Tail-Equivalent Linearization Method (TELM) is used. The genesis of the method lies in the first-order reliability method (FORM) [8] and the earlier works of Li and Der Kiureghian [16], Der Kiureghian [7] and Koo et al. [15]. This paper formalizes the method and investigates the various characteristics of the tail-equivalent linear system (TELS).

Briefly stated, in TELM, the stochastic excitation is discretized and represented in terms of a finite number of standard normal random variables. In the space of these random variables, the domain defining a given response threshold is linearized at the point of maximum likelihood. The linearized domain uniquely defines the TELS. Linear random vibration analysis with the TELS then yields the response statistics of interest for the specified threshold. In contrast to the linear system defined in the ELM, the TELS critically depends on the considered response threshold. Through this dependence, the TELM is able to provide a first-order approximation of the non-Gaussian distribution of the nonlinear response. Furthermore, the method provides good approximations of the mean up-crossing rate and first-passage probability of the nonlinear response, particularly for high thresholds. Although the TELM is more generally applicable, this paper mainly focuses on stationary response of non-degrading hysteretic structures subjected to a zero-mean Gaussian excitation.

After describing a method for discrete representation of the stochastic excitation, geometric characteristics of a linear system in the space of standard normal random variables are examined. It is shown that a reversible relationship exists between the impulse response function of the system and the gradient vector of a hyperplane defining a threshold of interest. This then leads to a formal definition of the TELS for a general nonlinear system. Issues related to the existence and uniqueness of the TELS and the influences of various key parameters on the TELS are examined. An algorithm for finding the sequence of linearization points necessary for determining the full probability distribution of the response is next described, followed by a discussion of methods for determining various response statistics. Throughout the paper results are presented for a hysteretic oscillator and, where appropriate, comparisons are made between results obtained by the TELM and the conventional ELM. At the end, a multi-degree-of-freedom hysteretic structure subjected to base motion is considered, for which results based on the TELM are compared with results obtained by the Monte Carlo simulation (MCS) method.

2. Discrete representation of stochastic excitation

An essential step in the TELM is the discrete representation of the input stochastic excitation in terms of a finite set of standard normal random variables. A number of formulations for this purpose are available (see [7] for a brief review). For a zero-mean, second-order (i.e., finite variance) Gaussian process $F(t)$, all representations have the form

$$F(t) = \sum_{i=1}^{n} s_i(t) u_i = \mathbf{s}(t)\,\mathbf{u}, \tag{1}$$

where $\mathbf{u} = [u_1, u_2, \ldots, u_n]^{\mathrm{T}}$ is a vector of standard normal random variables, $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_n(t)]$ is a row vector of deterministic basis functions dependent on the covariance structure of the excitation process, and n is a measure of the resolution of the representation. The main difference between the various formulations lies in the selection of the basis functions, $s_i(t)$. TELM can be developed in conjunction with any of these formulations. Here, a time-domain formulation that is a smoothed version of the filtered random pulse train described in Der Kiureghian [7] is used. This formulation is particularly convenient for earthquake engineering applications, which is the focus of numerical examples considered in this paper.

Let the process $F(t)$ be described as the output of a linear filter excited by a white noise, $W(t)$,

$$F(t) = \int_0^t h_f(t-\tau) W(\tau)\mathrm{d}\tau, \tag{2}$$

where $h_f(t)$ denotes the impulse-response function (IRF) of the filter. For a filter that is stable and has finite variance in response to white noise, the process $F(t)$ becomes stationary after a duration at which the IRF $h_f(t)$ diminishes to zero. Consider the sequence of equally spaced time points $t_i = t_{i-1} + \Delta t$, $i = 1, 2, \ldots, t_n$, with $t_0 = 0$ and Δt a small time step. For $0 \le t \le t_n$, we discretize the process $F(t)$ by replacing the white noise in (2) by an approximating rectangular wave process defined by

$$\hat{W}(t) = \frac{1}{\Delta t}\int_{t_{i-1}}^{t_i} W(\tau)\mathrm{d}\tau \quad t_{i-1} < t \le t_i, i = 1, 2, \ldots, n. \tag{3}$$

It is easy to show that the wave amplitudes $w_i = \hat{W}(t)$, $t_{i-1} < t \le t_i$, $i = 1, 2, \ldots, n$, are statistically independent normal random variables having zero means and the variance $\sigma^2 = 2\pi S/\Delta t$, where S is the spectral density of the white noise. Thus, the approximating process has a finite variance.