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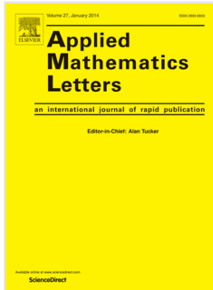

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Christopher S. Goodrich

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A CONVEXITY RESULT FOR FRACTIONAL DIFFERENCES

CHRISTOPHER S. GOODRICH

Abstract. In this brief note we demonstrate that under certain conditions the positivity of the fractional difference $\Delta_0^{\mu} y(t)$, for a given function $y : \mathbb{N}_0 \to \mathbb{R}$ and a number $\mu \in (2, 3)$, implies the convexity of y. This is given as a special case of a more general result. Finally, as a concrete application of this result we demonstrate that a particular class of fractional boundary value problems has no nontrivial positive solutions.

1. Introduction

The discrete fractional calculus has enjoyed a substantial development over the past several years. First spurred on by the seminal investigations of Atici and Eloe [1, 2], who developed some of the basic theoretical properties of the discrete fractional calculus, many additional studies have subsequently developed the application of the discrete fractional calculus to such problems as fractional boundary value problems (FBVPs) [3, 9, 12, 13, 15, 17, 18, 19, 20], initial value problems [2, 4], modeling [6], operational properties of fractional sums and differences [5, 10, 11, 16, 21], exponential functions in discrete fractional calculus [7], and the extension of the fractional calculus to more general time scales [8, 14].

In spite of this significant amount of initial research, there remain many open questions concerning the theory of discrete fractional calculus. A particularly unfortunate aspect of this concerns the lack of geometrical intuition. Unlike in the integer-order calculus, which is replete with geometrical meaning, the fractional calculus seems resistant to this sort of reasoning. Largely this is due to the definition of the fractional difference, which is very non-geometrical and, importantly, is nonlocal in character. Indeed, for a function $y : \mathbb{N}_0 \to \mathbb{R}$ and a number

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