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A stochastic analysis of a nonlinear flow response

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Abstract

In this article a stochastic analysis of velocity fluctuations from a nonlinear flow response is presented. The statistical analysis is based on probability averages of the flow quantities evaluated over several realizations of the examined turbulent flow. In order to define the realizations, a long-time record of a turbulent velocity signal is cut up into pieces of length T, where T is much longer than the characteristic velocity fluctuation correlation time occurring in the flow. These pieces are then treated as observations of different responses in an ensemble of similarly simulated flows. The ergodic assumption is investigated, and it is shown that in some regions of the flow the standard statistical approach of time average fails to capture the nonlinear response of the flow. The ergodicity deviations based on samples from three-dimensional numerical simulations are compared with theoretical predictions given by scaling arguments and also with data from experimental observations. A very good agreement is observed. Calculations of the normalized auto-correlation functions and power spectra are also performed.

Keywords: Turbulent flow; Velocity fluctuations; Ergodicity; Correlation time; Power spectrum

1. Introduction

In a general case a formal statistical treatment is based on probability averages evaluated over an ensemble of several realizations of the same process, which defines a stochastic set. For an ergodic process the probability average can be replaced by the conventional time average, and the statistical analysis is more feasible. Nevertheless, when the turbulence is dominated by large and coherent structures, typically strongly correlated, the ergodic hypothesis cannot be assumed and only a probability or statistical average (i.e. ensemble averages) should be used to describe the statistical quantities of the flow [1,2]. In a numerical simulation context, the total time of simulation needs to be long enough to ensure the ergodicity of the process or to lead a convergence of the statistics. In a multiple scale flow like turbulence this total time may be different for each region in the domain.

The purpose of this paper is to perform a statistical analysis of turbulent velocity fluctuations resulting from three-dimensional large eddy numerical simulations and experimental observations of the same flow explored in the simulations. In addition, the integral timescales and the ergodic character of the nonlinear flow response are evaluated. A scaling analysis is also developed in order to estimate the deviation ε between the time average and probability average associated with the ergodicity assumption. The nonlinear response of the flow striking a cube mounted over a flat plate is used to investigate the long time statistics in order to quantify the velocity fluctuation correlation time at different positions on the flow domain. The numerical and experimental results are confronted with the statistics given by a temporal analysis and the deviation between the two approaches characterized by the ergodic parameter, ε predicted by a scaling analysis. All statistical quantities are calculated using the probability average approach and the associated error bars are shown.

1.1. Scalings

The scales of the large eddies are set by the geometry and the speed of the stirring mechanism, while cut-off scales of the small eddies are determined by the action of viscosity. Here one concentrates on the small scales for a flow with large eddies of

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given velocity, length and time scales U_ℓ, ℓ, T_ℓ. The important Kolmogorov micro-scale for the smallest eddies is based on a further assumption that the smallest eddies depend only on the rate at which energy is put into the large eddies, i.e. on one particular combination of U_ℓ, ℓ. The friction only acts on the smallest scale and the energy is supplied only at the large scale. The rate of dissipation $\epsilon = 2\nu\overline{\mathbf{D}' : \mathbf{D}'}$ is measured per unit of mass, and can be related to the macro-scales by assuming that a significant fraction of the kinetic energy per unit of mass $k = (1/2)\overline{\mathbf{u}' \cdot \mathbf{u}'}$ in the large eddies is dissipated in the turnover time of the large eddies, i.e. per unit time

$$\rho\epsilon = \rho U_\ell^2/T_\ell, \quad \text{therefore } \epsilon = U_\ell^3/\ell. \tag{1}$$

Here $\mathbf{u}' = \mathbf{u} - \widetilde{\mathbf{u}}$ denotes the velocity fluctuations, $\mathbf{u}$ is the instantaneous velocity, $\widetilde{\mathbf{u}}$ is the average velocity and $\mathbf{D}'$ is the shear rate fluctuation tensor defined in terms of the velocity fluctuations, i.e. $2\mathbf{D}' = (\nabla\mathbf{u}' + \nabla(\mathbf{u}')^{\mathrm{T}})$, where the T denotes the transpose operation. Now, the dimensions of this dissipation per unit mass ϵ are L^2T^{-3}, while the dimensions of the kinematic viscosity ν are L^2T^{-1}. Hence by a simple dimensional analysis, we obtain the velocity, length, time and strain-rate scalings of the Kolmogorov micro-scale [3]: $U_k = (\nu\epsilon)^{1/4}$, $\ell_k = (\nu^3/\epsilon)^{1/4}$, $T_k = (\nu/\epsilon)^{1/2}$ and $S_k = (\epsilon/\nu)^{1/2}$. Introducing the Reynolds number of the large scale eddies $Re = U_\ell\ell/\nu$, one obtains

$$\begin{aligned} U_k/U_\ell &= Re^{-1/4}, \quad \ell_k = \ell Re^{-3/4}, \\ T_k &= T_\ell Re^{-1/2}, \quad S_k = S_\ell Re^{1/2}. \end{aligned} \tag{2}$$

The Kolmogorov micro-scale for the smallest eddies depends on the velocity and length scales of the large eddies in the combination $\epsilon = U_\ell/\ell$. Note the turnover time of the smallest eddies T_k is shorter than the turnover time of the large eddies T_ℓ by the fact $Re^{-1/2}$. Hence mixing takes place faster and more efficiently on small scales than on large scales. Large scale mixing however is described by the Taylor diffusivity $D = U_\ell\,\ell$. So, for a container of height H, the time for eddy diffusion is then $H^2/D = T_\ell H^2/\ell^2$ [4].

A second micro-scale, the Taylor micro-scale, uses a different combination to yield a slightly larger scale. The Taylor micro-scale λ_τ can be thought of as the boundary layer thickness on the edge of a large eddy, i.e.

$$\lambda_\tau = (\nu t)^{1/2} \tag{3}$$

with t being the turnover time of the large eddies $T_\ell = \ell/U_\ell$. Hence, using the Reynolds number of the large eddies we can show $\lambda_\tau = (\nu\ell/U_\ell)^{1/2} = \ell Re^{-1/2}$.

2. Statistical analysis

The main goal of this work is to treat statistically turbulent velocity signals both from numerical simulations and experimental observations. An important question addressed here is to look at how long a time average is necessary to obtain well converged statistical results. To this end, we have looked at the difference between the time average and an ensemble average as the measure of this convergence.

The flow is considered a stochastic process given by a family of functions $u = u(t, \alpha)$, where $\alpha = 1 \ldots N$ are the realizations of the process. For a stationary random process then, we may, in principle at least, determine the various probability distributions from the observations of $u(t)$ for one realization of the system over a long period of time. This long-time record can be cut up into pieces of length T (where T is much longer than any periodicities occurring in the process), and these pieces may be treated as observations of different realizations of the system in an ensemble of similarly prepared systems. The underlying assumption here is the so-called ergodic assumption [5], which states that for a stationary random process, a large number of observations made on a single system at N arbitrary instants of time have the same statistical properties as observing N arbitrarily chosen systems at the same time from an ensemble of similar systems. In dealing with a general stochastic process, there are two types of mean values that can be evaluated. One is the probability average obtained by a number sufficiently larger (N) of observations at some fixed time t, denoting this average by $\langle u(t)\rangle$, and the other is the time average made for a function $u(t)$, denoting this average by $\underline{u}(\alpha)$. In the case of a stationary ergodic random process, both averages yield the same result. A time average $\underline{u}(\alpha)$ over a sufficiently long realization α_o of the process is defined as [6]

$$\underline{u}(\alpha_o) = \lim_{T\to\infty} \frac{1}{T}\int_0^T u(t, \alpha_o)\mathrm{d}t. \tag{4}$$

For the method of averaging defined in equation Eq. (4) to have any significance, it is necessary that the limit exists and that it be independent of the definition of the time T. For a stationary random process, this condition is in fact satisfied.

Now, whether each realization has the same probability to occur, a statistical (or probability) average is defined as being [7]

$$\langle u(t)\rangle = \lim_{N\to\infty} \frac{1}{N}\sum_{\alpha=1}^{N} u(t, \alpha). \tag{5}$$

According to ergodic assumption the time average thereby obtained is the same with the probability average (5), provided that the function u is finite and continuous in mean-square [8].

A fluctuation about the probability average, is defined as being $u'(t) = u(t) - \langle u\rangle$. The variances (or the turbulent kinetic energy $(1/2)\langle u'^2\rangle$) are calculated from the probability average as well as the time average. We therefore use the following expression

$$\langle u'^2(t)\rangle = \lim_{N\to\infty} \frac{1}{N}\sum_{\alpha=1}^{N}[u(\alpha, t) - \langle u(t)\rangle]^2 \tag{6}$$

for calculating the probability average of the square of the velocity fluctuation. While the time average of the square of the velocity fluctuations is evaluated by

$$\underline{u'^2} = \lim_{T\to\infty} \frac{1}{T}\int_0^T u'^2(t)\mathrm{d}t. \tag{7}$$