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Recursive least squares estimation algorithm applied to a class of linear-in-parameters output error moving average systems☆

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ABSTRACT

This letter deals with the identification problem of a class of linear-in-parameters output error moving average systems. The difficulty of identification is that there exist some unknown variables in the information vector. By means of the auxiliary model identification idea, an auxiliary model based recursive least squares algorithm is developed for identifying the parameters of the proposed system. The simulation results confirm the conclusion.

1. Introduction

In recent years, various parameter estimation algorithms have been studied for linear systems [1–4] and nonlinear systems [5–7], and are important in signal processing and filtering [8,9], adaptive control [10] and system identification. Many identification methods have been reported, e.g., the errors-in-variables methods [11,12], the key term separation based methods [13,14]. The least squares methods can be roughly divided into two categories. One is the iterative methods for solving matrix equations [15] and for offline identification, and the other is the recursive methods for online identification. Recently, new least squares based parameter estimation algorithms have been widely explored for system identification. For example, Hu and Ding presented an iterative least squares estimation algorithm for controlled moving average systems based on matrix decomposition [16,17]; Li et al. proposed a maximum likelihood least squares identification method for Hammerstein finite impulse response moving average systems [18]; Wang et al. developed a data filtering based least square algorithm for Hammerstein systems [6] and CARARMA systems [19]; Ding proposed a novel coupled least squares algorithm for estimating the parameters of the multiple linear regression models [20].

The auxiliary model identification idea can be used for studying identification problems in the presence of some unknown variables in the information vector of the identification model [21]. Ding and Chen presented a dual-rate system identification method using the auxiliary model to estimate the unknown noise-free output of the system, and directly to identify the parameters of the underlying fast single rate model from dual-rate input–output data [22,23]. Ding proposed a combined state and least squares parameter estimation algorithm for canonical state space systems [24], a hierarchical multi-innovation stochastic gradient algorithm for Hammerstein nonlinear systems [25] and a two-stage least squares based iterative algorithm for CARARMA systems [26].

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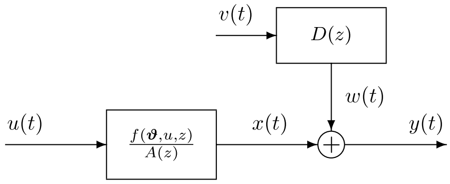

Fig. 1. A class of linear-in-parameters output error moving average systems.

The object of this letter is, by means of the auxiliary method, to derive an auxiliary model based recursive least squares algorithm for identifying a class of linear-in-parameters output error moving average systems based on the available input–output data.

This letter is organized as follows. Section 2 introduces a class of linear-in-parameters output error moving average systems and derives its identification model. Section 3 proposes an auxiliary model based recursive least squares algorithm. Section 4 provides an illustrative example. Finally, conclusions are given in Section 5.

2. Problem formulation

Let us define some symbols. The symbol $\boldsymbol{I}_n$ denotes an identity matrix of order n; $\mathbf{1}_n$ denotes an n-dimensional column vector whose elements are 1; the superscript T denotes the matrix/vector transpose.

Consider the following stochastic system shown in Fig. 1 [1]:

$$y(t) = \frac{f(\boldsymbol{\vartheta}, u(t), z)}{A(z)} + D(z)v(t), \tag{1}$$

where $\{u(t)\}$ is the system input sequence, $\{y(t)\}$ is the system output sequence, $\{v(t)\}$ is the stochastic white noise with zero mean and variance σ^2, $A(z)$ and $D(z)$ are polynomials, of known orders (n_a, n_d), in the unit backward shift operator z^{-1}, and defined by

$$A(z) := 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{n_a} z^{-n_a} \in \mathbb{R},$$
$$D(z) := 1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_{n_d} z^{-n_d} \in \mathbb{R},$$

and $w(t) := D(z)v(t)$ is referred to as a moving average process. Assume that $f(\boldsymbol{\vartheta}, u(t), z)$ is a linear function with the parameter vector $\boldsymbol{\vartheta} \in \mathbb{R}^{n_b}$, and can be expressed in a least squares form as

$$f(\boldsymbol{\vartheta}, u(t), z) := \boldsymbol{\vartheta}^{\mathrm{T}} \boldsymbol{\eta}(u(t), u(t-1), \ldots, u(t-n_b+1)),$$

which is denoted simply by

$$f(\boldsymbol{\vartheta}, u(t), z) = \boldsymbol{\vartheta}^{\mathrm{T}} \boldsymbol{\eta}(u(t), z, n_b),$$

where $\boldsymbol{\eta}(u(t), z, n_b) := \boldsymbol{\eta}(u(t), u(t-1), \ldots, u(t-n_b+1))$ is the linear or nonlinear vector of $u(t), u(t-1), \ldots, u(t-n_b+1)$. In this case, the system in (1) can be equivalently written as [27]

$$y(t) = \frac{\boldsymbol{\vartheta}^{\mathrm{T}} \boldsymbol{\eta}(u(t), z, n_b)}{A(z)} + D(z)v(t). \tag{2}$$

Without loss of generality, assume that $u(t) = 0, y(t) = 0$ and $v(t) = 0$ as $t \leqslant 0$.

Remark 1. It is worth noting that the systems in (2) may be linear systems or nonlinear systems. For example, for the linear system

$$y(t) = \frac{B(z)}{A(z)} u(t) + D(z)v(t),$$
$$B(z) := b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{n_b} z^{-n_b}, \tag{3}$$

we let $\boldsymbol{\vartheta} := [b_1, b_2, \ldots, b_{n_b}]^{\mathrm{T}} \in \mathbb{R}^{n_b}$ and $\boldsymbol{\eta}(u(t), z, n_b) := [u(t), u(t-1), \ldots, u(t-n_b+1)]^{\mathrm{T}}$, and have $f(\boldsymbol{\vartheta}, u(t), z) = B(z)u(t) = \boldsymbol{\vartheta}^{\mathrm{T}} \boldsymbol{\eta}(u(t), z, n_b)$. For

$$f(\boldsymbol{\vartheta}, u(t), z) = \boldsymbol{\vartheta}_1 \sin(u(t)) + \boldsymbol{\vartheta}_2 \cos^2(u(t-1)) + \cdots + \boldsymbol{\vartheta}_{n_b} \sqrt[3]{u^2(t-n_b+1)}.$$

Eq. (1) is a nonlinear system but is a linear-in-parameters system. Notice that as $D(z) = 1$, system (3) reduces to an output error system.

In this letter, we intend to develop a new identification algorithm for estimating the parameter vector $\boldsymbol{\vartheta}$ and the parameters of $A(z)$ and $D(z)$ in model (1) by utilizing measured input–output data $\{u(t), y(t) : t = 1, 2, \ldots\}$.