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Random field theory to interpret the spatial variability of lacustrine soils

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The mechanical characterisation of heterogeneous soils, such as alluvial deposits, is commonly performed through a deterministic approach. This latter consists on applying the engineering judgment to choose a mean trend from continuous vertical readings of in field investigations (e.g. Cone Penetration Tests – CPTs) or discontinuous ones as Standard Penetration Tests (SPTs). Conversely, in order to take into account the spatial variability of mechanical measurements of the soil the spatial standard deviation, the scale of fluctuation and the autocorrelation function have to be calculated. This latter approach follows the stochastic field theory and it can be fruitfully applied to all those soil formations that are inherently heterogeneous. In this paper, theoretical bases of this approach has been briefly described and a practical application to lacustrine soil deposits at Popoli site located in Abruzzi Region (Italy) are presented. The methods introduced are not straightforward but they provide information that can be used to improve both the reliability of the geotechnical design and the efficiency of the soil use depending on the investigated depths and the measurement intervals.

1. Introduction

The spatial variability of natural soil deposits alongside depth could be responsible for the lack of serviceability in greenhouses, factories, farms, silos and other types of rural and urban structures. Quantifying this source of uncertainty in shallow or deep foundation design contributes to increase the designing reliability and the efficiency in their management (e.g. Cherubini & Vessia, 2010; Lesny & Paikowsky, 2011, pp. 47–65). At the same time, in precision agriculture, management zones (MZs) (Yao et al., 2014) have been introduced to identify homogeneous zones within crop fields in order to classify and delimit contiguous areas aimed at site-specific management (Córdoba, Bruno, Costa, Peralta, & Balzarini, 2016 and references here in). This latter application uses several statistical and geostatistical techniques to interpolate spatial data although, as in the footing design applications the spatial correlation and autocorrelation structures of those soil properties who guide the zonation activity shall be provided with. Within the framework of the random field theory (Vanmarcke, 1984) these variables whose values fluctuate in space can be considered as "random variables": they vary in

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space according to both a deterministic trend based on physical laws and random spatial variations due to the soil depositional evolution. Thus, in order to study the spatial fluctuations of physical, chemical and mechanical properties at first, spatial continuous measurements (e.g. profiles or records) should be performed. Then, the de-trended measurement profiles can be studied within the theoretical framework of the random field theory. It is provided with a set of functions and parameters enabling several different applications related to the management of the spatial inherent variation of natural soil measurements retrieved through in field testing. In this paper at first, the concepts of the trend function, the scale of fluctuation, the point variance and the variance reduction function and the autocorrelation function have been introduced to characterise the spatial inherent variability of soil properties. Then, some techniques are proposed to calculate the aforementioned functions and parameters for the case study of the tip resistance readings from mechanical cone penetration tests CPTs performed in lacustrine deposits placed in Abruzzi region (Italy). section 2 introduces the random field theory used to characterise the spatial variability of natural soils. section 3 discusses the application of the preceding approach to four readings of tip resistance Q_c from CPTs performed at Popoli site (PE). The proposed spatial variability structure can be applied to geotechnical designing, litotechnical surface characterisation as well as all those agricultural applications and land use management where the spatial variability of natural soil parameters must be assessed.

2. Main features of the random field theory

Vanmarcke (1984) postulated that spatial variation of soil hydro-physical-mechanical variables can be characterised by other than only one value or a mean trend measured through continuous or discontinuous readings. As a matter of fact, the spatial waving values of a continuous profile of measures *m* can be divided into a mean trend *t* and a fluctuation or residual profile values ε according to the following relation (Vanmarcke, 1977):

$$m(z) = t(z) + \varepsilon(z) \quad (1)$$

t(z) is commonly represented by an interpolation function selected through the mean square root (LSM) method. When its determination coefficient R^2 is quite high (higher than 0.6) the de-trended measures, named fluctuations, will be randomly distributed about the zero value and their summation alongside depth will be approximately zero. They can be defined as random field. A random field is a range of random numbers whose indices are identified with a continuous set of points in the space. A random field is spatially correlated meaning that adjacent values do not differ as much as values that are further apart.

In order to use the fluctuation profiles as one dimensional random fields they must satisfy the stationarity condition in weak-sense. This means that only the first two moments (the mean and the variance) of ε(z) are required to be constant: that is also named *second-order stationarity*. To check the weak stationarity, two useful techniques are suggested in literature, that are: (1) the modified Bartlett test (Phoon, Quek, & An, 2003) and (2) the Cusum test (Caulcutt, 1983). Further details on these methods and their applications are described in Cherubini, Vessia, and Pula (2007); a brief description of Cusum procedure is given in section 3 where this method has been applied to the measurements under study.

From ε(z) profile the fluctuation scale δ can be drawn. This latter measures the distance within which the ε(z) values show a strong spatial correlation. It is analytically expressed by the formula:

$$\delta = 2\int_0^\infty \rho(z)dz \quad (2)$$

where ρ(z) is the autocorrelation function for a distance z. This latter function represents the shape of spatial dependency of measures falling in a spatial interval δ. All these quantities enables to characterise the variability structure of a spatial random variable whose dimension can be one, two or three; it will be called 1D, 2D or 3D random field. Soil parameters for agricultural and geotechnical purposes are commonly measured along horizontal and vertical directions. When a spatial characterisation is undertaken, several vertical readings are recorded at narrow or large distance depending on the needs of land use management or the extension and the importance of the structures to be built.

From a set of fluctuations and their spatial distribution along vertical direction (1D random field) the scale of fluctuation can be derived through several methods. These methods can be divided into two groups: graphical (Rice, 1944; Vanmarcke, 1984) and analytical (Baecher & Christian, 2003; Lloret-Cabot, Fenton, & Hicks, 2014). The graphical approaches derive δ from the distances between two subsequent crossings of the zero-fluctuation axis (Fig. 1).

As the graphical methods are concerned, named d_i the distance between two adjacent crossings whose width is detected by the operator, the scale of fluctuation can be calculated through Vanmarcke's rule according to the formula:

$$\delta = \sqrt{\frac{2}{\pi}} mean(d_i) \quad (3)$$

The graphical approach gives the advantage of the straightforward estimate of the scale of fluctuation directly from the visual insight of fluctuations. The knowledge of δ allows to reduce the amount of the variance σ^2 to the averaged spatial variance $\sigma^2_{averaged}$ through the variance reduction function Γ^2:

$$\sigma^2_{averaged} = \Gamma^2 \cdot \sigma^2 \quad (4)$$

In its simplest shape, it can be assumed (Vanmarcke, 1984) as:

$$\Gamma^2 = \frac{\delta}{L}\left(1 - exp\left(-\frac{L}{\delta}\right)\right) \quad if(L/\delta) > 1 \quad (5)$$

where L is the spatial interval of interest. According to the application, L can be the length of a pile shaft, of a structure to be designed, of a depth of interest for cultivations.

The scale of fluctuation can be calculated also by analytical methods. These methods calculate at first the autocorrelation