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journal homepage: www.elsevier.com/locate/issn/15375110

Special Issue: Numerical tools for soils

Research Paper

Multifractal and joint multifractal analysis of general soil properties and altitude along a transect

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ARTICLE INFO

Article history:
Published online xxx

Keywords:
Spatial variability
Soil general properties
Scaling heterogeneity
Multifractals
Joint multifractals

Multifractal characterisation of soil spatial variability has the potential for providing a better understanding of the distribution patterns of data values, and may contribute to improved resource management. We examined the scaling heterogeneity and multiple scale relationships of soil general properties and topography using multifractal and joint multifractal techniques. Soil samples were collected down to 0.20 m depth and altitude was recorded at equal intervals of 3 m along a 396 m transect in an Orthic Podzol at Pernambuco, Brazil. Soil properties studied were: textural fractions, pH, organic carbon (OC), exchangeable cations, exchangeable acidity (H + Al), sum of bases (SB), cation exchange capacity (CEC) and percent base saturation (V). The spatial distribution of altitude and soil general properties, characterised through generalised dimension, D_q, and singularity spectra, $f(\alpha)-\alpha$, showed a well-defined multifractal structure. Notwithstanding, these variables displayed several degrees of scaling heterogeneity, which was lowest for pH, sand and clay contents and highest for exchangeable cations and silt content. Joint multifractals showed that correlations between pairs of variables may or may not be stronger at the observation scale than across a range of spatial scales. Hence, soil OC and pH showed higher relationships to CEC, (H + Al), SB and V at the observation than at multiple spatial scales, while local topography effects on pH and CEC were greater at multiple scales. Multifractal and joint multifractal analysis provided new insights to characterise the spatial patterns and the relationships between soil properties at multiple scales, and to evaluate the effect of topography on soil heterogeneity.

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https://doi.org/10.1016/j.biosystemseng.2017.08.024

Please cite this article in press as: Siqueira, G. M., et al., Multifractal and joint multifractal analysis of general soil properties and altitude along a transect, *Biosystems Engineering* (2017), https://doi.org/10.1016/j.biosystemseng.2017.08.024

1. Introduction

Soil is considered as a natural body of variable depth, differentiated into horizons with complex mineral and organic composition, usually unconsolidated, which differs from the parent material below in morphology, and in physical, chemical and biological properties. In natural soils, composition, properties and functions depend upon soil forming factors (climate, parent material, topography, time, and vegetation) that vary both spatially and seasonally. In agricultural soils, a variety of additional factors (including tillage system, fertilisation and crop management, among others) may affect soil properties and processes, and as a result, they may enhance or reduce natural variability associated with soil forming factors. Therefore, many soil properties and attributes, including moisture, texture, organic carbon content, acidity, properties of the exchange complex, available nutrients, etc., have been shown to exhibit spatial variability (Caridad-Cancela, Vidal Vázquez, Vieira, Abreu, & Paz González, 2005; Morales, Vázquez, & Paz-Ferreiro, 2011; Paz, Taboada, & Gómez, 1996; Paz González, Vieira, & Taboada Castro, 2000; Vieira, Hatfield, Nielsen, & Biggar, 1983).

Several methods can be employed to characterise the structure of the spatial dependence of a variable and to map their respective variability. Spatial variability of soil properties has been analysed most frequently using geostatistical methods (McBratney & Webster, 1981; Morales et al., 2011; Vieira, de Carvalho, Ceddia, & González, 2010). However, the geostatistical approach is only able to address how the second moment of a variable change with scales or frequencies. Furthermore, a second order moment can only provide a weak characterisation of the variability that occurs in non-normal distributions (Biswas, Cresswell, & Si, 2012; Zeleke & Si, 2005). Spatial variability in soils frequently cannot be described by a normal distribution and in addition it may exhibit periodicity, nonstationarity, nonlinearity and many other characteristics. Thus, a complete characterisation of soil spatial variability distributions other than normal, demands higher moment orders (Caniego, Espejo, Martín, & San José, 2005; Vidal Vázquez, Miranda, & Paz González, 2005; Vidal-Vázquez et al., 2013; Zeleke & Si, 2004, 2006).

Fractal theory has been used in the past to quantitatively characterise spatial variability from geological, soil and environmental systems over a large range of measurement scales. According to the fractal theory, the properties that are observed at different scales are related to each other by a power function (self-similarity), whose exponent is called the fractal dimension. Power law relationships observed at different scales have been shown to be useful to describe scale invariance in various fields of earth sciences, including soil science (Pachepsky & Crawford, 2004; Vidal Vázquez et al., 2005; Pachepsky & Hill, 2017). The fractal approach needs a single scaling exponent to describe the statistical distribution of a data set. However, heterogeneity and variability of soil properties can occur in different intensities at different scales, and therefore a single fractal dimension may not be sufficient to describe the scaling relationships.

Multifractal analysis implies that a statistically self-similar measure can be represented as a combination of multiple fractal dimensions with corresponding scaling exponents. Multifractal behaviour of several soil properties have been reported since the 1990s (Grout, Tarquis, & Wiesner, 1998; Pachepsky & Hill, 2017; Paz-Ferreiro, Miranda, & Vidal Vázquez, 2010). In particular, the multifractal approach has proven to be useful to deal with spatial variability of soil properties from samples taken along horizontal transects (Caniego et al., 2005; Zeleke & Si, 2005, 2006; Ji et al., 2016; López de Herrera, Herrero Tejedor, Saa-Requejo, & Tarquis, 2016; Vidal-Vázquez et al., 2013; Wang, Shu, Xie, Liu, & Si, 2011, 2009) or vertical profiles (Marinho, Pereira, Vidal Vázquez, Lado, & Paz González, 2017; Siqueira, Silva, Montenegro, Vidal Vázquez, & Paz-Ferreiro, 2013; Wilson, Mirás Avalos, Lado, & Paz-González, 2016).

While multifractal analysis alludes to the distribution of a single variable along its support, joint multifractal analysis refers to the joint distribution of two or more variables with a common spatial or temporal support. Joint multifractals have been employed to characterise the joint distribution of paired variables with individual multifractal behaviour in several fields including agronomy (Kravchenko, Bullock, & Boast, 2000) soil science (Zeleke & Si, 2004) and environmental science (Jiménez-Hornero, Jiménez-Hornero, Gutiérrez de Ravé, & Pavón-Domínguez, 2010). Moreover, joint multifractals have been used for simultaneous analysis of three multifractal measures determined over the same spatial or temporal support (Pavón-Domínguez, Jiménez-Hornero, & Gutiérrez de Ravé, 2015).

Simultaneous multifractal and joint multifractal analysis has been used to characterise the spatial patterns of variability of soil data sets sampled either on one dimensional (Ji et al., 2016; Wang et al., 2011, 2009; Zeleke & Si, 2005, 2006), or two dimensional (Zhang et al., 2013) supports. Similarly, multifractal and joint multifractal analysis has been used to analyse the spatial heterogeneity of crop yield and slope at the field scale (Kravchenko et al., 2000) and that of leaf area index (LAI) and topographic variables along a 381 m transect (Banerjee, He, Guo, & Si, 2011).

Thus, the joint multifractal approach has been previously employed to investigate links between several paired soil properties at multiple scales For example, Zeleke and Si (2005, 2006) and Wang, Shu, Liu, and Si (2009), focused on joint multifractal analysis of soil texture fractions and water content or water retention, while Zhang et al. (2013) addressed that of selected soil nutrients. However, until now, there is little information on multiple scale relationships between interacting and multifractally distributed soil general properties. Besides, joint multifractal analysis of local topography and soil properties has been not performed previously.

Brazil is the leading country in sugar cane (*Saccharum officinarum* sp.) production. The State of Pernambuco located in the northeast region has the 5th highest sugarcane production of the country. The lowlands coastal region of this state was formerly covered by humid Atlantic forest (Mata Atlântica). Already, since the beginning of the colonial period, in the sixteenth century, this area has been devoted to sugarcane plantations, whose extent has increased over the centuries. Multifractal and joint multifractal techniques may be useful to understand the association among soil variables and between soil variables topography across multiple scales in these

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