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# An extended optimal replacement model for a deteriorating system with inspections

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ABSTRACT

This study considers a generalized replacement model for a deteriorating system in which failures can only be detected by inspection. The system is assumed to have two types of failures and is replaced at the occurrence of the $N$th type I failure (minor failure), or the first type II failure (catastrophic failure), or at working age $T$, whichever occurs first. The probability of a type I or type II failure depends on the number of type I failures since the previous replacement. Such a system can be repaired after a type I failure, but is deteriorating stochastically. That is, the operating intervals are decreasing stochastically, whereas the durations of the repairs are increasing stochastically. Based on these assumptions, we determine the expected net cost rate and discuss various special cases of the model. Finally, we develop a computational algorithm for finding the optimal policy and present a numerical example to show the properties of the proposed model.



## 1. Introduction

Most replacement models assume that the repaired system or machine is returned to an “as good as new” state (or called perfect repair). This assumption means that, upon repair, an item has the same life distribution as a new item. If we only consider operating time with a negligible repair time, a renewal process is obtained. If we consider the non-zero repair times which are independent and identically distributed (i.i.d.) random variables, then an alternating renewal process with up and down periods is obtained. However, for a repairable deteriorating system, it may be more realistic to assume that a failed item will return to an “as bad as old” state after being repaired. That is, if the life distribution of a new item was $F$, then the item after repair will have the survival function $\overline{F}_t$, where $t$ is the item's age at failure and $\overline{F}_t(x)=\overline{F}(t+x)/\overline{F}(t)$. This is also known as minimal repair (see [1,2] for more detail). Brown and Proschan [3] examined an imperfect repair that has probability $p$ of being a perfect repair and probability $1-p$ of being a minimal repair. Their model was generalized by Block et al. [4] to the case in which the probability of a perfect repair is age-dependent. These results were further generalized by Sheu and Griffith [5,6] to the multi-item case. For more advanced practical applications using minimal repair models, Chang et al. [7] proposed a multi-criteria optimal replacement policy for a system subject to shocks. Xia et al. [8] studied dynamic maintenance decision-making based on a developed Multi-Attribute Model and Maintenance Time Window (MAM-MTW) methodology, and Xia et al. [9] studied the optimized maintenance schedule for energy systems subject to degradation through the MAM.

However, for some practical deteriorating systems, the system can be different from those described above. Lam [10] considered a machine maintenance problem where after each repair, the machine's operating time becomes stochastically shorter. On the other hand, to model the effect of aging, repair times become stochastically larger. Therefore, we consider a repair and replacement model with such a feature for analyzing deteriorating systems. Due to the non-increasing operating times after repairs, the system will eventually die out, so the total system life is finite. One possible way of modeling this kind of deteriorating systems is to use the non-homogeneous Poisson process (NHPP). Baxter [11], Gupta and Kirmani [12], and Kochar [13] showed that, if the failure rate function is non-decreasing

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(non-increasing) in the NHPP, the operating intervals are stochastically non-increasing (non-decreasing). Lam [10] formulated a geometric process replacement model, in which the successive operating intervals, $\{X_i,\ i=1,2,\ldots\}$, of an item constitute a non-increasing geometric process, and the consecutive repair durations, $\{Y_i,\ i=1,2,\ldots\}$, constitute a non-decreasing geometric process.

To reduce the chances of a failure during the system's operation, Barlow and Hunter [14] considered the periodic system replacement or overhaul at times $T$, $2T$, … (for some $T>0$), combined with a minimal repair at the system failure. This model was generalized by Beichelt [15], Boland and Proschan [16], Boland [17], Block et al. [18], Sheu et al. [19], and Sheu [20,21], among others. Makabe and Morimura [22–24] proposed a replacement model in which a system is replaced at the $N$th failure, and also discussed the optimal policy for this situation. This model was generalized by Morimura [25], Nakagawa [26], and Sheu and Griffith [27]. These authors assumed that a failed system becomes functional after a minimal repair and used the NHPP to model these deteriorating systems.

Rangan and Esther Grace [28] considered the optimal replacement policies for a deteriorating system with imperfect maintenance. Lam [29] and Stadje and Zuckerman [30] studied monotone process replacement models in which the repair times were also not negligible. These models were generalized by Sheu [31].

Maintenance and replacement models are usually based on the assumption that failures are detected and repaired simultaneously. However, for some systems, failures may be only identified by inspection. Christer and Waller [32] proposed a basic inspection maintenance model. Baohe [33] studied an optimal inspection and diagnosis policy for a system with self-announcing and non-self-announcing modes. Wang [34] considered two types of inspections in a delay-time-based setting. Flage [35] studied a delay time model with imperfect and failure-inducing inspections. Lee and Song [36] considered the system reliability updating of fatigue-induced sequential failures through inspection and repair. Nakagawa et al. [37] summarized the policies for periodic and random inspection. Chen et al. [38] applied periodic and random inspection policies to computer systems. Zhao et al. [39] studied optimal time and random inspection policies for computer systems. Optimal periodic and random inspection with first, last, and overtime policies are proposed in Zhao and Nakagawa [40]. Cheng and Li [41] studied a geometric process repair model with inspections. They assumed that a system failure is detected by inspection. In their study, they only considered one failure mode in two common and predominant types of systems. However, the model with one failure mode may not be realistic for many practical systems. Hence, in this study, we develop a model with multiple failure modes with a replacement condition for the system's working age. Our model has potential applications in oil and gas industries, medicine, and nuclear power plants. There are two types of failures, both of which can only be detected by inspection. The system is replaced at the $N$th type I failure (minor failure), first type II failure (catastrophic failure), or at working age $T$, whichever occurs first. The probability of a type I or II failure depends on the number of failures since the last replacement. Our model is a generalization of the models of Lam [10,29], Rangan and Esther Grace [28], Sheu [31] and Cheng and Li [41]. The formulation and optimization of the model is presented in Section 2. Some special cases are discussed in Section 3. A numerical example is provided in Section 4 and concluding remarks are made in Section 5.

## 2. System description and model formulation

The following symbols are used.

| | |
|---|---|
| $N$ | the number of type I failures occurs where the system is replaced |
| $T$ | the working age of the system |
| $M$ | the number of failures until the first type II failure occurs |
| $\overline{P}_k$ | the probability that the first $k$ failures occur are type I failures; $\overline{P}_k=P(M>k)$; $1=\overline{P}_0\geq\overline{P}_1\geq\cdots$ |
| $\{\overline{P}_k\}$ | the abbreviation for a sequence of probabilities |
| $q_k$ | Pr{a type I failure occurs when the system fails}; $q_k=\overline{P}_k/\overline{P}_{k-1}$ |
| $\theta_k$ | Pr{a type II failure occurs when the system fails}; $\theta_k=1-q_k$ |
| $X_i$ | the operating interval of the system after the $(i-1)$th repair |
| $H_i(\cdot)$ | c.d.f. (cumulative distribution function) of random variable $X_i$ |
| $E[X_i]=\lambda_i$ | the mean of the operating interval after the $(i-1)$th repair |
| $Y_i$ | the repair duration after the detection of the $i$th failure |
| $E[Y_i]=\mu_i$ | the mean of the repair duration after the $i$th failure detected |
| $\gamma^{i-1}h$ | the time period between two successive inspections in the $i$th repair cycle which we denote that the time interval between completion of the $(i-1)$th failure repair and the completion of the $i$th failure repair of the system is called the $i$th repair cycle of the system |
| $h$ | the time period between two successive inspections in the first repair cycle and is a real number greater than zero |
| $\gamma$ | a constant and its value is $0<\gamma<1$ to adjust the inspections for the operating interval accordingly |
| $G_i$ | the idle period of the system when the $i$th failure occurs until it is detected |
| $E[G_i]$ | the mean idle period of the system when the $i$th failure occurs until it is detected |
| $Q_{(k,1)}$ | the number of inspections during the operating period of the system after the $(K-1)$th repair if the system is finally replaced at the $N$th type I failure or at the first type II failure |
| $E[Q_{(k,1)}]$ | the mean value of $Q_{(k,1)}$ |
| $Q_{(k,2)}$ | the number of inspections during the operating period of the system after the $(k-1)$th repair if the system is finally replaced at the working age $T$ |
| $A_{(l)}$ | denote the number of inspections where the working age of the system is $l$ |
| $E[Q_{(k,2)}]$ | the mean value of $Q_{(k,2)}$ |
| $C_1$ | the repair cost rate |
| $C_2$ | the reward rate whenever the system is operating |
| $C_3$ | the replacement cost |
| $C_4$ | the penalty cost rate for the idle period |