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A new class of Wiener process models for degradation analysis

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ABSTRACT

For many products, it is not uncommon to see that a unit with a higher degradation rate has a more volatile degradation path. Motivated by this observation, we propose a new class of random effects model for the Wiener process model. We express the Wiener process in a special form and allow one of the parameters to be random across the product population so that a unit with a high degradation rate would also possess high volatility. Statistical inference of the model is discussed. By the same token, we introduce a stress–acceleration relation for the Wiener process so that both the degradation rate and the volatility of the product are increasing in the stress level. The proposed models are demonstrated by analyzing a dataset of fatigue crack growth and a dataset of head wears of hard disk drives. The applications suggest that our models perform better than existing models that ignore the positive correlation between the drift rate and the volatility.

1. Introduction

For some products such as expensive equipment or highly reliable devices, product failure data are scarce. The lack of failure data makes the prediction of failure time distribution and the subsequent decisions, such as warranty design and preventive maintenance scheduling challenging tasks. It is found that most ageing failures can be attributed to some underlying degradation mechanism, under which the damage accumulates over time and eventually leads to a product failure when the accumulated damage reaches a failure threshold, either random or specified by industrial standards. For example, it has been a common practice to define the lifetime of a light emitting diode (LED) as the time when the lumen output of the LED lighting first crosses the threshold line of 70% of its initial lumen output level. The corresponding lifetime is often written as L70. Similar examples are capacity drop in batteries, gearbox vibration, tyre wear, increase in resistance for electronic devices such as electromagnetic relays. The degradation-threshold failure phenomenon provides an intimate link between degradation and product failures. The failure time distribution and the parameters therein can be determined through analysis of the degradation mechanism and the degradation data collected from this product. The degradation model can then be used in burn-in modelling [33] and maintenance optimization [3,13]. A famous degradation model is the Wiener process with positive drifts, where the first-passage-time of its degradation path to a fixed failure threshold level follows an inverse Gaussian distribution. A popular representation of the Wiener process in degradation analysis is

$$X(t)=\nu\Lambda(t)+\sigma\mathcal{B}(\Lambda(t)), \quad (1)$$

where ν is the drift parameter reflecting the rate of degradation, σ is the volatility parameter, $\mathcal{B}(\cdot)$ is the standard Brownian motion, and $\Lambda(\cdot)$ is a monotone increasing function representing a general time scale [31]. By convention, we let $\Lambda(0)=0$ and $X(0)=0$.

The basic Wiener process (1) has received wide applications in degradation analysis. As some examples, Whitmore and Schenkelberg [31] used it to model resistance increase in a self-regulating heating cable; Le Son et al. [8] found a good fit of (1) to the 2008 PHM Conference Challenge data. Hu et al. [4] applied the same model to an LED dataset. Nevertheless, many real applications suggest that degradation of a batch of products is usually affected by two types of variability, i.e., individual variability and temporal variability. The temporal variability is the inherent variation in the degradation for a unit; while the individual variability determines heterogeneity among the degradation paths of different product units. This variability refers to the phenomenon that under the same operating and environmental conditions, the observed degradation in a product population is very different due to some unobservable effects, such as variations in the raw materials and in the manufacturing processes. The basic model (1) is capable of capturing the inherent variability. Nevertheless, it fails to model the individual variability. In the literature, the individual variability has been well modelled by introducing random effects into (1). A random-effects model modifies certain parameters of the

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degradation model to be unit-specific and they follow a certain distribution. For example, Peng and Tseng [18] proposed a Wiener process with random effects model. They assumed that different units in the population have different realizations of the drift parameter ν, while they share the same volatility parameter σ. Typically, a normal distribution for ν yields an analytically tractable random effects model. This model was investigated in-depth by Peng and Tseng [18], and was found a good fit to the famous GaAs laser degradation data in Meeker and Escobar [14, Example 13.5]. Si et al. [23] adopted the same random-drift Wiener process model to analyze gyroscopic drift data in an inertial navigation platform used in weapon systems, and a fatigue-crack length growth dataset of the 2017-T4 aluminum alloy. More applications of this model can be found in Tsai et al. [25], Si et al. [22], Wang et al. [30,28], etc. Similar idea of imposing a normal distribution on ν was also extensively analyzed from the Bayesian perspective. See Feng et al. [2], Jin et al. [6], Liao and Tian [11], etc., for some recent references and Jardine et al. [5], Si et al. [21] for reviews of the related literature. Recently, Peng and Tseng [20] proposed a more general random-drift model by assuming a skew-normal distribution for ν, which includes the normal distribution and the truncated normal as special cases.

The above random-drift Wiener process models assume a constant σ across the product population. In reality, however, it is not uncommon to see that when the degradation of a product unit is faster, the variation of the degradation over time is also higher. This means that a unit with a larger drift parameter ν is expected to have a larger volatility parameter σ as well. In the famous GaAs laser degradation data example presented in Meeker and Escobar [14, Example 13.5], if we fit each of the 15 degradation paths with (1) by letting $\Lambda(t)=t$, we can obtain 15 pairs of $(\hat{\nu}_i, \hat{\sigma}_i)$. A simple calculation shows that the sample correlation coefficient of $\hat{\nu}$ and $\hat{\sigma}$ is 0.487. The positive correlation between the mean and the variance of the degradation path can be easily captured when the Gamma process model or the inverse Gaussian process model is used. For example, the mean and the variance of the homogeneous Gamma process $\{Y(t), t>0\}$, where $Y(t) \sim \text{Gamma}(at, b)$, are at/b and at/b^2, respectively [36]. A unit with a higher degradation rate, i.e., either a large a or a small b, will also have a high volatility. The same relation holds for the inverse Gaussian process, where $Y(t) \sim \mathcal{IG}(at, bt^2)$ and the mean and the variance of $Y(t)$ are, respectively, at and a^3t/b [35]. Nevertheless, the current Wiener process models available in the literature fail to capture this important relation. Wang [27] introduced a random effects model by letting $\sigma^{-2} \sim \text{Gamma}(r, \delta)$ and $[\nu|\sigma^2] \sim \mathcal{N}(\mu, \theta\sigma^2)$. Based on this random effects model, a larger realization of σ would result in a larger variation in ν rather than a larger ν. Therefore, this model fails to capture the positive correlation between ν and σ. In this paper, we introduce a new class of random effects model that is able to model this positive correlation.

Even when the degradation is quite homogeneous under the same operating conditions, e.g., for products produced from a mature manufacturing line, it is expected that degradation of a product would be hastened when the product is operated/tested under more severe conditions. The operating conditions, when observable, can be treated as covariates and incorporated into the degradation process. The covariates are also called stresses in the literature of accelerated degradation testing (e.g., [19]). Incorporation of covariates in the Wiener process (1) has been well studied in the literature. Doksum and Normand [1] used the Wiener process to describe a biomarker series and they assumed that ν is a function of the covariates while σ being a constant. This assumption is also adopted in Park and Padgett [16,17], Liao and Tseng [9], Lim and Yum [12]. However, similar to the analysis for the random effects models above, we expect that when the degradation rate increases, the degradation variation would also become larger in some applications. Whitmore and Schenkelberg [31] fitted the degradation data of each individual unit separately and then used linear regression to establish the relationship between ν and the transformed stress as well as the relationship between σ and the stress. Their analysis revealed that both ν and σ are increasing in the testing temperature. Liao and Elsayed [10] also assumed that both ν and σ in (1) are increasing functions of the stress. If we assume that ν and σ are independent functions of the stresses, however, there might be excessive parameters to be estimated. To reduce the number of parameters, Peng and Tseng [19] incorporate the covariates using the cumulative exposure model. Basically, the cumulative exposure model assumes that operation under a stress for a duration of t is equivalent to the operation under a baseline stress for a duration of $\rho_s \cdot t$, where ρ_s is a scaling factor depending on the stress s. In this work, we propose another approach to incorporate the stresses so that both ν and σ are increasing functions of the stress. The idea is similar to the random effects model proposed in this study.

The rest of the paper is organized as follows. Section 2 introduces the new random effects Wiener process model such that the degradation rate changes along with the degradation volatility of a unit. The EM algorithm is used for the statistical inference and its performance is evaluated through simulation. Section 3 presents the stress–acceleration relation for the Wiener process as a counterpart of the new random effects model. Section 4 demonstrates the random effects model using the fatigue crack growth data reported in Wu and Ni [32] and the laser data in Meeker and Escobar [14, Example 13.5], The stress–acceleration model is demonstrated using a new head wear dataset of hard disk drives (HDDs). Section 5 concludes the paper with discussions.

2. A random effects model

To introduce the new random effects model, we express the Wiener process as

$$X(t) = \upsilon\Lambda(t) + \varsigma\upsilon\mathcal{B}(\Lambda(t)). \tag{2}$$

Compared it with (1), it is easily seen that $\sigma = \varsigma\upsilon$. However, the new expression (2) enables us to introduce correlation between υ and σ. In the proposed random-effects model, the parameter υ is treated as random across the population. A unit with a high realization of υ would possess a high degradation rate and a high variation in the degradation path, which is more realistic in many applications. A tractable random effects model results when we assume $\eta = 1/\upsilon \sim \mathcal{N}(\mu, \omega^2)$. As argued in the Introduction, the normal assumption for the random effects has been widely adopted for the Wiener process (1). This partially supports our assumption here. In fact, when the probability of a negative η is small, say, $\mu - 3\omega > 0$, a negative realization of υ is unlikely and the normal assumption would be meaningful. By integrating out the random effects, it is readily shown that the unconditional distribution of $X(t)$ is

$$f_{X(t)}(x) = \frac{(x\omega^2 + \mu\varsigma^2)\Lambda(t)}{\sqrt{2\pi[x^2\omega^2 + \varsigma^2\Lambda(t)]^3}} \exp\left(-\frac{(\mu x - \Lambda(t))^2}{2(\varsigma^2\Lambda(t) + \omega^2 x^2)}\right).$$

We shall highlight that if the negative probability is significant, we may use the truncated normal distribution for η instead, with support on $[a, \infty)$, $a > 0$. The unconditional distribution of $X(t)$ also has a closed-form, though more complicated. Statistical inferences under the truncated-normal random-effects are similar to the normal case. For simplicity, we use the normal distribution for η throughout the paper for demonstration.

In practice, the realization of υ for a particular unit is fixed yet unknown and unobservable. In prognostics and health management,