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Environmental stress screening modelling, analysis and optimization

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ABSTRACT

Environmental stress screening (ESS) is widely used in industry to eliminate early or latent failures. However, the appropriate stochastic modeling for ESS has not been yet suggested in the literature. In this paper, we develop the corresponding stochastic model and analyze the effect of the ESS in terms of population characteristics. In our model, during the ESS, the manufactured items are exposed to a stress with the relatively large magnitude, which can result either in immediate failures of items or in the increased susceptibility to future failures. The corresponding optimization problems for obtaining the optimal level of the stress magnitude are also formulated and discussed. An illustrative example is considered.

1. Introduction

The failure rate of manufactured items often initially decreases, which is usually called the “infant mortality”. The most popular explanation of this phenomenon is that a population is a mixture of weak items (i.e., items with shorter lifetimes) and strong ones (i.e., items with longer lifetimes). As the “weakest populations are dying out first”, this can result in the initially decreasing FR. In order to eliminate weak items and, therefore, to improve the ‘quality’ of manufactured items a burn-in procedure is usually employed in practice (see, e.g., Mi [1–3], Cha [4], Yun et al. [5], Wu et al. [6], Kim and Kuo [7], Kim [8], Cha and Finkelstein [9]). Thus, the ‘sufficient condition’ for employing the *traditional* burn-in is that the items should have the initially decreasing failure rate.

However, more thorough analysis (e.g., Fiorentino and Saari [10], Yan and English [11]) of mixed populations indicates that weak subpopulations can also result from the latent defects that cause *additional* failure modes. In this case, the FR does not necessarily initially decrease (see Example 1 in Cha and Finkelstein [12]) and, therefore, the traditional burn-in procedure is not effective at all. On the other hand, another procedure that is usually referred to as the environmental stress screening (ESS) can help in this case. During the ESS, a short-time excessive stress is applied to eliminate weak items with latent defects. Thus, distinct from burn-in, the ESS can be understood as the method of elimination of items with “additional failure modes”. Moreover, as it was mentioned, the ESS does not require the initially decreasing FR. The formal difference between the ESS and burn-in was only recently clarified in our previous work (see Cha and Finkelstein [12]). Although there have been some empirical studies on the ESS in the literature, Cha and Finkelstein [12] is the first work to deal with adequate stochastic modelling, analysis and optimization of the ESS. Therefore, we feel that the second paper dealing with a different stochastic model is quite appropriate and justified.

In the current paper, we develop a *new stochastic model for the ESS*, where an external shock can either destroy an item with a given probability or increase the ‘defect size’ of the defective item by a random amount. Our previous paper (Cha and Finkelstein [12]) is based on the ‘extreme shock model’ (there is no wear (degradation) in the paper), whereas here we employ direct competing risks framework considering gradual degradation effect of the system due to increasing defect size. Thus, the stochastic model for ESS in the current paper is different in essence and can be applied to, e.g., solder joints, where defective items have microcracks which gradually grow as the items experience external shocks and cause the failure of items when they reach a threshold level.

The paper is organized as follows. In Section 2 we discuss general supplementary results for extreme-type and ‘combined’ shock models to be extensively used in the forthcoming sections.

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Acronyms

Cdf	cumulative distribution function
FR	instantaneous failure rate (function)
pdf	probability density function
r.v.	random variable
Sf	survivor function

Notation

$\{N(t),\ t\geq 0\}$	a nonhomogeneous Poisson process of shocks
$\lambda(t)$	the intensity function of the nonhomogeneous Poisson process of shocks
$T_i,\ i=1,2,\ldots$	sequential arrival time of the nonhomogeneous Poisson process; a r.v.
$p(t)$	the probability of immediate system failure on a shock occurred at time t; $q(t)=1-p(t)$
$\{N_q(t),\ t\geq 0\}$	the 'thinned' original process with the thinning probability $q(t)$
$W_i, i=1,2,\ldots$	the random increment of wear at i th shock; a r.v.
$M_W(\cdot)$	the moment generating function of W_i's
$W(t)$	the random wear of the system at time t (given no critical shock has occurred until time t)
R	random threshold level which defines the system failure due to the accumulated wear
T_N	lifetime of a non-defective item; a r.v.
$r(t)$	baseline FR of the system in the absence of shocks
W_M	the initial wear; a r.v.
T_E	the lifetime in the field use that accounts only for the external shock failure mode of defective items (i.e., the lifetime without any other causes of failure)
T	the lifetime of the system before ESS; a r.v.
$\lambda_T(t)$	the FR of T
s	the magnitude of the stress imposed during the ESS
s^*	the optimal magnitude of the stress
$\alpha(s)$	the probability of immediate system failure during the ESS (the function of s)
$\rho(s)$	the proportion of non-defective items which becomes defective due to the ESS (the function of s)
T_{ESS}	the population distribution in field use after the ESS; a r.v.
$\lambda_E(t,s)$	the FR of T_{ESS} as the function of s
$\overline{F}_E(t,s)$	the Sf of T_{ESS} as the function of s
$\pi,\ 1-\pi$	the proportion of the non-defective and the defective items, respectively
τ	the mission time of an item in field operation
$M(s)$	the mean time to failure of an item in field operation as a function of s
$c(s)$	the total expected cost
c_{sr}	the shop replacement cost
c_s	the cost for conducting the ESS
K	the gain given when the mission (of length τ) is successful
C	the penalty given when the mission (of length τ) is not successful

Section 3 is devoted to the detailed probabilistic analysis of the suggested model. In Section 4, we discuss and illustrate the corresponding optimization problem. Finally, some concluding remarks are given in the last section.

2. Preliminaries

In this section, we discuss some general supplementary results on shocks modeling that will be extensively used for the description and analysis of the ESS model in the rest of this paper. These preliminaries are mostly based on our previous work (Cha and Finkelstein [13–14]).

Denote by $\{N(t),\ t\geq 0\}$ the nonhomogeneous Poisson process (NHPP) of shocks with rate $\lambda(t)$ and arrival times $T_i,\ i=1,2,\ldots$. Assume that this shock process is the only cause of failures for the system and that each shock results in the system's failure with probability $p(t)$ and has no effect on a system with probability $q(t)=1-p(t)$.

Denote the corresponding time to failure of the system by T_S. It is well-known (see, e.g., Finkelstein [15] and references therein) that the survival probability of the system in this model, which is often called in the literature *the extreme shock model,* is

$$P(T_S>t)\equiv\overline{F}_S(t)=\exp\left(-\int_0^t p(u)\,\lambda(u)du\right), \tag{1}$$

whereas the corresponding FR $\lambda_S(t)$ is

$$\lambda_S(t)=p(t)\lambda(t),\quad t\geq 0. \tag{2}$$

For convenience, in what follows we will refer to this shock model as the '$p(t)\Leftrightarrow q(t)$ model'.

Assume now that the ith shock is critical (i.e., it results in an immediate failure) with probability $p(T_i)$ and with probability $q(T_i)=1-p(T_i)$, it increases the wear of a system by a random increment $W_i\geq 0$ (non-critical shock). In accordance with this setting, the random wear of the system at time t is (given no critical shock has occurred until time t)

$$W(t)=\sum_{i=0}^{N_q(t)} W_i,$$

where $N_q(t)$ is the 'thinned' original process with the thinning probability $q(t)$, representing the total number of non-critical shocks in $(0,t]$, and, formally, $W_0=0$ corresponds to the case $N_q(t)=0$ when there are no shocks in $(0,t]$. Failure occurs when a critical shock occurs or $W(t)$ reaches the random boundary R. Therefore, denoting by $E_C(t)$ the event that no critical shock has occurred until time t,

$$\begin{aligned}P(T_S\geq t\,|\,&N(s),0\leq s\leq t;\ \ W_1,W_2,\ldots,W_{N(t)};R)\\&=P(E_C(t)\,|\,N(s),0\leq s\leq t;\ \ W_1,W_2,\ldots,W_{N(t)};R)\\&\quad\times P(W(t)\leq R\,|\,N(s),0\leq s\leq t;\ \ W_1,W_2,\ldots,W_{N(t)};\ \ R;E_C(t))\\&=\prod_{i=0}^{N(t)} q(T_i)I\left(\sum_{i=0}^{N(t)} W_i\leq R\right),\end{aligned} \tag{3}$$

where $q(T_0)\equiv 1$.

Eq. (3) is very general and certain assumption should be made in order to integrate out the corresponding uncertainties and to obtain the analytically tractable solution. Therefore, let $W_i, i=1,2,\ldots$ be the i.i.d. r.v.'s and R be exponentially distributed with parameter θ. Then $P(T_S\geq t)$ can be obtained explicitly by direct derivation (Theorem 1 of Cha and Finkelstein [13]) as

$$\begin{aligned}P(T_S\geq t)&=E\left[\left(\prod_{i=0}^{N(t)} q(T_i)\right)\cdot\exp\left\{-\theta\sum\nolimits_{i=0}^{N(t)} W_i\right\}\right]\\&=\exp\left\{-\int_0^t (p(x)+q(x)(1-M_W(-\theta)))\lambda(x)dx\right\},t\geq 0,\end{aligned}$$