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First and second order approximate reliability analysis methods using evidence theory

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ABSTRACT

The first order approximate reliability method (FARM) and second order approximate reliability method (SARM) are formulated based on evidence theory in this paper. The proposed methods can significantly improve the computational efficiency for evidence-theory-based reliability analysis, while generally provide sufficient precision. First, the most probable focal element (MPFE), an important concept as the most probable point (MPP) in probability-theory-based reliability analysis, is searched using a uniformity approach. Subsequently, FARM approximates the limit-state function around the MPFE using the linear Taylor series, while SARM approximates it using the quadratic Taylor series. With the first and second order approximations, the reliability interval composed of the belief measure and the plausibility measure is efficiently obtained for FARM and SARM, respectively. Two simple problems with explicit expressions and one engineering application of vehicle frontal impact are presented to demonstrate the effectiveness of the proposed methods.

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1. Introduction

Uncertainties widely exist in practical engineering problems, which should be appropriately quantified and controlled for the reliability and safety of a product [1,2]. Usually, uncertainties can be classified into two distinct types: aleatory and epistemic uncertainties [3–5]. Aleatory uncertainty describes the inherent variation associated with a physical system or environment, which is often dealt with probability theory [6–9]. Epistemic uncertainty refers to the lack of information or data in some phases of the modeling process, which, therefore, can be reduced with the collection of more information. The probability theory has been traditionally used to model epistemic uncertainty, generally by picking some familiar probability distribution and its associated parameters to represent one's belief in the likelihood of possible values. However, for some distributions (e.g. normal or weibull), even small epistemic uncertainty in probability distribution parameters can cause large changes in the tails of the distributions, which may result in unnegligible influence on the reliability analysis results for practical engineering problems [10].

Evidence theory was proposed and developed by Dempster and Shafer [11,12], which provides a promising supplement to probability theory for representation of epistemic uncertainty [13]. First, evidence theory employs a much more flexible framework with respect to the body of evidence and its measures. For example, when information is enough to construct parameter probability distributions, evidence theory can provide an equivalent description to probability theory model. Second, evidence theory can deal directly with situations in which both aleatory and epistemic uncertainties exist. This capability is important because the available data in many engineering problems commonly contain both interval-valued information (epistemic uncertainty) and probability distributions (aleatory uncertainty). Third, evidence theory does not require the assumption of input probability distributions when there is a lack of information.

Due to the above advantages, evidence theory has recently been applied in structural reliability analysis and design. Some exploratory work in this area has been reported, which can be classified into several main categories:

(i) *Comparison between evidence theory and the other uncertainty analysis models*. Several methods for obtaining the evidence theory and probability boxes structures were introduced in [14], which shows that these two structures can be inter-convertible. Probability theory, evidence theory, possibility theory and interval analysis were explored and compared in uncertainty representation and propagation with some benchmark problems [15]. Evidence theory and Bayesian theory were compared in uncertainty modeling and decision making, which indicates that Bayesian probabilities can help make a decision when there is considerable uncertainty [16].

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(ii) *Reliability analysis*. An evidence theory model considering dependence between parameters was formulated for the structural reliability analysis [17]. A structural reliability analysis method using evidence theory was developed by introducing a non-probabilistic reliability index approach [18]. By integrating the moment concept and finite element method, a static and dynamic response analysis approach was formulated for structures with epistemic uncertainty [19]. A sampling-based approach [20] and a semi-analytic approach [21] were developed for sensitivity analysis of the uncertainty propagation problems using evidence theory.

(iii) *Reliability based design optimization*. A design optimization method was developed to handle the mixed epistemic and random uncertainties, in which the vicinity of the optimal point and the active constraints are quickly identified and hence a high computational efficiency is achieved [22]. An evidence-theory-based multidisciplinary design optimization method was proposed for structures with epistemic uncertainty through a sequential approximate strategy [23]. Based on combined probability and evidence theory, a reliability-based multidisciplinary design optimization approach was proposed for the mixed aleatory and epistemic uncertainties [24].

Though some important progresses were achieved above, evidence theory was barely used in practical engineering applications. One main reason is the high computational cost [25]. In evidence-theory-based reliability analysis, the uncertainty is propagated through a discrete probability assignment due to limited information, which is generally described by a series of discontinuous sets rather than an explicit continuous function like the probability density function in probability theory. Thus, time-consuming uncertainty analyses are required among each set for the assessment of its contribution to the reliability, which will inevitably result in expensive computational cost for a multidimensional problem when using evidence theory to conduct the reliability analysis. Aiming at this issue, several numerical methods [25–27] have been proposed to improve the computational efficiency, mainly by introducing the response surface technique. However, the precision of these methods is not usually stable since the response surface is influenced by many factors such as the selection of sampling techniques and approximation model types, and so on. As discussed above, actually a close relationship exists between evidence theory and probability theory, and that is why in many cases evidence theory is also called “imprecise probability”. It then seems natural and also reasonable that some important concepts or well-established techniques in traditional probabilistic reliability analysis could be introduced into the evidence-theory-based reliability analysis, based on which a series of effective reliability methods might be developed.

In this paper, the first and second order approximate reliability methods are proposed for evidence theory, which can significantly improve the computational efficiency of evidence-theory-based reliability analysis. The remainder of this paper is organized as follows. The conventional reliability analysis using evidence theory is introduced in Section 2. The first order approximate reliability method (FARM) and second order approximate reliability method (SARM) are formulated in Section 3. Three numerical examples are investigated in Section 4. Finally conclusions are summarized in Section 5.

2. Conventional reliability analysis using evidence theory

Consider the following reliability analysis problem:

$$g(\mathbf{X}) = g_0 \tag{1}$$

where $\mathbf{X}$ is a vector of n independent uncertain input parameters and they are modeled by the evidence variables in this paper; $g(\mathbf{X})$ is the limit-state function which is usually used to describe the safety or failure state of a structure; g_0 denotes an allowable value of the limit-state function. The safety region G for this problem is defined as:

$$G = \{\mathbf{X} | g(\mathbf{X}) \geq g_0\} \tag{2}$$

The conventional reliability analysis using evidence theory is illustrated with the above simple example, which includes two main steps: the construction of joint basic probability assignment and the computation of reliability interval.

2.1. Construction of joint basic probability assignment

Evidence-theory-based reliability analysis starts by defining a frame of discernment (FD) that is a set of mutually exclusive elementary subsets for each evidence variable X, which is similar to the sample space in probability theory. The FDs for all evidence variables in a problem form the uncertainty domain. In this paper, The FD is also denoted as X. All the possible values of the FD will form a power set $\Omega(X)$.

After defining the FD, the basic probability assignment (BPA) that represents the degree of belief is assigned to each subset of the FD power set based on the statistical data or the expert experience. The BPA is assigned through a mapping function: $\Omega(X) \to [0, 1]$ which should satisfy the following three axioms:

$$m(A) \geq 0 \quad \text{for any } A \in \Omega(X) \tag{3}$$

$$m(\varnothing) = 0 \tag{4}$$

$$\sum_{A \in \Omega(X)} m(A) = 1 \tag{5}$$

where $m(A)$ refers to the degree of belief that is assigned to the subset A. In this paper, we assume that the subsets A are all closed intervals. Each set $A \in \Omega(X)$ satisfying $m(A) > 0$ is called the focal element. Sometimes the information available for a parameter may come from different sources, thus, the evidence should be aggregated by rules of combination [12,28,29].

Similar to the joint probability density function in probability theory, the joint basic probability assignment should be constructed in evidence theory when multiple uncertain variables are involved. Due to the independence among the parameters, the joint basic probability assignment m can be obtained for an n-dimensional problem as below:

$$m(A) = \begin{cases} \prod_{i=1}^{n} m_i(A_i) & \text{when } A_i \in \Omega(X_i),\ i = 1, 2, ..., n \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where A_i and $\Omega(X_i)$ are the focal element and FD power set of the parameter X_i, respectively, and A is the focal element of the Cartesian Product Θ, which can be defined as follows:

$$\begin{aligned} \Theta &= \Omega(X_1) \times \Omega(X_2) \cdots \times \Omega(X_j) \cdots \times \Omega(X_n) \\ &= \{A = [A_1, A_2, \ldots, A_i, \ldots, A_n], A_i \in \Omega(X_i), \quad i = 1, 2, \ldots, n\}, \quad 1 \leq j \leq n \end{aligned} \tag{7}$$

2.2. Computation of reliability interval

It should be pointed out that evidence theory employs an interval composed of the belief measure (Bel) and the plausibility measure (Pl) to characterize uncertainty of the structural response, rather than a single measure in probability theory.

Based on the obtained joint BPA and the given safety region, the reliability interval $[Bel(G), Pl(G)]$ of the safety event $X \in G$ for the above example can be calculated as below:

$$Bel(G) = \sum_{A \subseteq G} m(A) \tag{8}$$

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