Nonlinear Analysis: Hybrid Systems 31 (2019) 1–18

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Variable impulsive consensus of nonlinear multi-agent systems

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ARTICLE INFO

Article history:
Received 19 December 2017
Accepted 25 July 2018
Available online xxxx

Keywords:
Variable impulsive control
Consensus
Multi-agent systems
Distributed control

ABSTRACT

In this paper, the consensus problem for nonlinear multi-agent systems with variable impulsive control method is studied. In order to decrease the communication wastage, a novel distributed impulsive protocol is designed to achieve consensus. Compared with the common impulsive consensus method with fixed impulsive instants, the variable impulsive consensus method proposed in this paper is more flexible and reliable in practical application. Based on Lyapunov stability theory and some inequality techniques, several novel impulsive consensus conditions are obtained to realize the consensus of multi-agent systems. Finally, some necessary simulations are performed to validate the effectiveness of theoretical results.

1. Introduction

Consensus of multi-agent systems has been extensively investigated in recent years due to its potential application in physics science and mathematics [1–6]. Generally, there are two research directions of consensus problem: the leaderless regulation problem and the leader-following tracking problem. For the leaderless case, the distributed controllers are designed for each node (agent), so that all nodes eventually converge to an unprescribed common value, which may be a constant, or may be time-varying [7–10]. For the tracking problem, a leader node is considered and acts as a command generator that generates the desired reference trajectory and ignores information from follower nodes [11–14].

There are many approaches to realize consensus of multi-agent systems, such as robust control [15,16], feedback control [17,18], adaptive control [19,20], etc. Compared with the above control algorithms, impulsive control can reduce the transmission waste in some cases, and the state information is only transmitted at impulsive instants, which dramatically reduces the amount of synchronization information transmitted among the nodes of multi-agent systems and makes the method more efficient in a large number of real-life applications [21–26].

In the literature about the impulsive consensus of multi-agent systems, some significant topics have been discussed, including convergence speed [27,28], cooperative tracking problem [29,30], consensus schemes with switching topologies [31,32], uncertainties [33,34] and control gain error [35–37], etc. Note that the proposed control schemes in the aforementioned literature are performed with fixed impulsive instants. However, due to the hardware constraints, real systems cannot put impulsive instants at expected time exactly. For instance, we want to impose an impulse at time instant τ, but the practical impulse maybe occur in a short time window $[\tau - r, \tau + r]$, where τ and r are the center and radius of the impulsive time window respectively. Obviously, this case is not accordant with the theoretical consensus condition for fixed impulsive

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https://doi.org/10.1016/j.nahs.2018.07.004
1751-570X/

control, which causes the failure of the consensus. To overcome this problem, the variable impulsive control strategy is an effective and necessary tool, which obtains larger control region in practical application. In recent years, some pioneering works investigated the stability, stabilization and synchronization with impulsive time windows, such as stability of Cohen–Grossberg neural networks [38], hybrid neural networks [39], stochastic fuzzy delayed neural networks [40], linear delayed impulsive differential systems [41] and impulsive functional differential systems [42], stabilization and synchronization of linear systems [43], general nonlinear systems [44] and Hopfield-type neural networks [45], synchronization of coupled delayed switched neural networks [46], periodically multiple state-jumps impulsive control systems [47], comparison system approach of impulsive control system [48], sandwich control systems (cyclic control system) [49]. However, due to the complexity of distributed variable impulsive control and communication graph (network topology), there are few works combining the consensus of multi-agent systems with impulsive time windows.

The impulsive consensus method with impulsive time window can relieve the shortage of fixed impulsive control, and the impulsive instant can be chosen within a fixed region (i.e., the impulsive time window). In fact, there is also another impulsive protocol (so-called odd impulsive control and its extension method) has the same function. That is to say, the consensus can be realized when the impulsive sequence $\{t_1, t_3, t_5, \ldots\}$ or $\{t_{2k-1}\}$ ($k = 1, 2, \ldots$) is determined, and there is not any restriction for the even impulsive sequence $\{t_2, t_4, t_6, \ldots\}$ or $\{t_{2k}\}$. In this case, we can choose impulsive instant t_{2k} within (t_{2k-1}, t_{2k+1}). It is worth noting that the extension of odd impulsive control strategy to variable impulsive consensus of multi-agent systems is meaningful and significant in real application.

Motivated by the above discussions, this paper studies the consensus of multi-agent systems with variable impulsive control. By designing an effective distributed impulsive controller, some novel sufficient conditions with impulsive time window and odd impulsive sequence are obtained. Due to the difficulty for computers and machines to put impulses at exact time, the existing control methods for consensus of multi-agent systems with fixed impulses are no longer practical and valid. Indeed, larger networks correspond to more distributed impulsive control nodes, and the common fixed impulsive control cannot meet the actual requirement. Thus, the variable impulsive control seems particularly necessary in the consensus scheme. In this paper, the variable impulsive consensus method with impulsive time window and odd impulsive sequence are investigated respectively. To the best of our knowledge, this is the first time to intensively explore the variable impulsive consensus with impulsive time window and odd impulsive sequence. The derived results are novel and practical for the consensus problem, and also very coincident with the real world.

The main contributions of this paper can be summarized as follows.

(1) The impulsive consensus of a very general class of multi-agent nonlinear systems with impulsive time window is studied for the first time. The restriction of fixed impulsive instants is changed into the interval of the centers or left endpoints of adjacent impulsive time window.
(2) The odd impulsive consensus of multi-agent nonlinear systems is studied for the first time. The restriction of fixed even impulsive instants is removed.
(3) Conditions for consensus are given in terms of simple algebraic inequalities, and impulsive control parameters (control gain and some impulsive instants) are analyzed and seen to depend on the graph and system parameters. This gives design guidance for selection of the controller parameters.
(4) Compared with the existing fixed impulsive consensus cases, the proposed variable impulsive control schemes can allow larger control region, which is more flexible and practical in real application.

The rest of the paper is organized as follows. Some preliminaries are described in the next section. In Section 3, the model of nonlinear multi-agent system is given. In Section 4, impulsive consensus of multi-agent system with impulsive time window is analyzed. The conditions of odd impulsive sequence are obtained in Section 5. In Section 6, the simulation results are presented to verify the effectiveness of consensus conditions and show the difference of several control methods. Finally, we conclude this paper in Section 7.

Throughout this paper, the superscript 'T' stands for the transpose for a matrix. $\mathbb{R}$, $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$ denote the real numbers, the n-dimensional Euclidean space, the set of all $n \times m$ real matrices respectively. Matrices, if not explicitly stated, are assumed to have compatible dimensions. Let $\mathbb{N}_+ = \{1, 2, \ldots\}$. I_n is the n dimensional identity matrix. $\mathrm{diag}(d_1, \ldots, d_N)$ indicates the diagonal matrix with diagonal elements d_1 to d_N. $\otimes$ denotes the Kronecker product. $\lambda_{\max}(A)$ denotes the maximal eigenvalue of matrix A.

2. Graph theory notions

Throughout this paper, the communication graph among the agents is represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with a nonempty finite set of N nodes $\mathcal{V} = \{v_1, \ldots, v_N\}$, a set of edges or arcs $\mathcal{E} \in (\mathcal{V} \times \mathcal{V})$, and the associate adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$. An edge rooted at node j and ended at node i is denoted by (v_j, v_i) and $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, otherwise $a_{ij} = 0$. We assume that there are no repeated edges and no self-loops, i.e., $a_{ii} = 0$, $\forall i \in \mathbb{N}_+$. Node j is called a neighbor of node i if $(v_j, v_i) \in \mathcal{E}$. The set of neighbors of node i is denoted as $N_i = \{j|(v_j, v_i) \in \mathcal{E}\}$. Define the in-degree of node i is $d_i = \sum_{j=1}^{N} a_{ij}$ and in-degree matrix as $D = \mathrm{diag}\{d_i\} \in \mathbb{R}^{N\times N}$. The Laplacian matrix is $L = D - \mathcal{A}$. In a directed graph, a sequence of successive edges in the form $\{(v_i, v_k), (v_k, v_l), \ldots, (v_m, v_j)\}$ is a direct path from node i to node j. A digraph is said to have a spanning tree, if there is a node (called a root node), such that there is a directed path from the root to any other node in the graph.