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Quantized feedback stabilization of continuous time-delay systems subject to actuator saturation

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ABSTRACT

In this paper, the problem of quantized feedback stabilization is investigated for continuous time-delay systems subject to actuator saturation. By utilizing two different methods, delay-independent conditions are obtained to guarantee the existence of a region of admissible initial conditions from which all trajectories of the resulting closed-loop system converge to a neighborhood of the equilibrium. Furthermore, delay-dependent conditions are developed based on the delay partitioning idea and the Lyapunov–Razumikhin functional approach, respectively. Finally, several examples are provided to demonstrate the effectiveness of the proposed approaches.

1. Introduction

Actuator saturation arises naturally in many engineering systems. It may cause loss in performance and even instability of the closed-loop system if it is ignored in the design process. In the last few years, many researchers have investigated control system analysis and design with actuator saturation. Moreover, general systematic methods based on rigorous theory were developed (see [1–3] and [4,5]). Stability analysis and stabilization problems of delay systems have attracted much attention during the past few years (see [6–13]). When time delay and actuator saturation are present in a control system, it is important to study the problems of stability and stabilization (for instance [14–17] and [18,19]). The design of controllers such that an estimate of the domain of attraction is as large as possible was considered in [14] and [16]. The state feedback and output feedback semi-global stabilization problems were studied in [18,20], respectively, while an algebraic Riccati equations (ARE) approach was developed in [20]. A state feedback controller was designed to guarantee that there exists a region of admissible initial conditions from which all trajectories of the resulting closed-loop system converge to some bounded region ([15] and [17]). Quantized control with delay was investigated in [15], where delay-dependent conditions were proposed. The problem of robust exponential stabilization was presented, and delay-independent conditions were reported in [17]. In [21], the delay partitioning idea was introduced to study the stability of continuous systems. Recently, a new inequality which was the modified version of free-matrix-based integral inequality was derived in [22], and then improved delay-dependent stability criteria which guaranteed the asymptotic stability of the system were presented. It is worth noting that there is not an intensive literature in the problems of quantized feedback stabilization for time-delay systems with input saturation.

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Quantized feedback has found applications in many engineering systems including digital control systems and networked systems. It rises to a challenging problem in the analysis and synthesis of control systems. Various types of quantizations have been investigated and a great number of results on quantized feedback systems have been reported in the literature (see [23–32] and the references therein). Practical compressive sensing systems were investigated in [33], where two approaches to sparse signal recovery in the face of saturation error were developed. Accordingly, the problem of quantized feedback controller design still has not been studied in [33]. The networked control system with quantization and actuator saturation was discussed in [34]. However, the more complicated situation with time-delay has not been dealt with in [34].

In [35], a framework based on a kind of Lyapunov approach and an alternative stabilization method based on the chaotic behavior of piecewise affine maps were provided, respectively. Moreover, the performances of these methods for dealing with scalar linear systems have been compared in [35]. An optimal quantized feedback has been studied in [36–38], where the corresponding optimal quantizers were designed, respectively. The input-to-state stabilization (ISS) has been extended to l_2 stabilization for linear systems with quantized feedback in [39]. The problem of stabilization for discrete-time linear systems via logarithmic quantized feedback was addressed in [23,30,40], where the method based on Tsypkin-type Lyapunov functions was applied in [23]. Also, many quantized feedback stabilization results for linear systems have been extended to nonlinear systems ([41] and [42]). Very recently, the problem of ISS with respect to external disturbances of control systems using measurements from a dynamic quantizer was solved in [43]. It is observed that the quantized feedback control problems for systems with logarithmic quantizer have been studied by many authors. However, the corresponding control problems of finite-range quantizer have been proposed so little.

Furthermore, one should note that the stabilization problem studied in [15] involves a linear system and a saturated quantizer. In the current paper, input saturation is present so that the considered system is a nonlinear one. Motivated by [15], we stabilize a nonlinear time-delay system by quantized feedback controls based on a finite-range quantizer. However, no input saturation is considered in [15]. In our paper, both input saturation and saturated quantizer are considered, thus the results in [15] are not applicable to the present system. Compared with [15], the delay partitioning technique is adopted in this paper, which is a less conservative method.

A continuous-time linear system with input saturation and quantized control law was considered in [44]. The contribution of [44] can be viewed as complementary to the results developed in [15], where the method developed in [44] provided convex optimization procedure to characterize the domain of attraction. In particular, the uniform quantizer presented in [44] does not have a finite number of quantization levels. In practice, quantizers have a finite dynamic range due to the voltage limits of hardware devices. Hence, there is a strong desire to use a finite number of quantization levels to represent the set of quantized values. Unlike the quantizer in [44], a finite-range quantizer with saturation level is addressed in this paper. Note that neither finite level quantization nor time delays are considered in [44]. Therefore, the results in [44] cannot be applied to the system considered in the current paper. Accordingly, we need to deal with the nonlinearity issues arising from actuator saturation and saturated quantizer. To the best of our knowledge, there is so far no result on domain of attraction estimation for time-delay systems with actuator saturation and saturated quantizer. In this paper, the corresponding results are developed by using a unique approach which is confirmed in the analysis of systems subject to nested saturations. Our aim here is to initiate the study in this area by utilizing an approach for the analysis of systems with nested saturations.

We highlight a number of key features:

- Sufficient conditions of quantized feedback stabilization for a class of continuous systems with both time-varying delay and actuator saturation are given;
- A finite-range quantizer with saturation level is considered;
- Two different techniques on how to design the quantized feedback controllers are developed.

In this paper, we study control systems whose input variables are quantized. The designed quantized feedback controllers can guarantee that all trajectories of the closed-loop system will converge to a smaller region for every initial condition from the admissible domain. The delay-independent and delay-dependent estimates of the corresponding domain are provided via the linear matrix inequality (LMI) scheme, respectively.

Notation. Throughout this paper, the following notations will be used. For real symmetric matrices M and N, the notation $M \geq N$ (respectively, $M > N$) means that the matrix $M - N$ is positive semi-definite (respectively, positive definite). I_n and $0_{m,n}$ represent the $n \times n$ identity matrix and $m \times n$ zero matrix. 1_m denotes a vector of dimension m with components equal to 1. For a real vector u, $u_{(i)}$ denotes the ith component of vector u. $\|u\|$ denotes its Euclidean vector norm. $\mathcal{C}_{n,d} = \mathcal{C}([-d, 0], \mathbb{R}^n)$ stands for the Banach space of continuous vector functions mapping the interval $[-d, 0]$ into $\mathbb{R}^n$. Furthermore, we define $\text{Sym}\{A\} = A + A^T$ and $\mathfrak{L}(H, U_1) = \{v \in \mathbb{R}^n : \|H_i v\| \leq U_{1(i)}\}$, where H_i is the ith row of the matrix H. For a number $\rho^{-1} > 0$ and a matrix $P \in \mathbb{R}^{n\times n}$, $P > 0$, the ellipsoid $\varepsilon(P, \rho^{-1})$ is defined by $\varepsilon(P, \rho^{-1}) = \{v \in \mathbb{R}^n : v^T P v \leq \rho^{-1}\}$.

2. Problem formulation

Consider a continuous system with a time delay in state and input saturation, which is described by

$$\dot{x}(t) = Ax(t) + A_d x(t - \tau(t)) + B\text{sat}_{U_1}(u(t)), \tag{1}$$