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Synchronization analysis of stochastic coupled systems with time delay on networks by periodically intermittent control and graph-theoretic method

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ABSTRACT

In this paper, the issue of inner exponential synchronization (IES) for a class of stochastic coupled systems with time-varying delay on networks (SCSTVDNs) is investigated. A periodically intermittent control strategy is introduced to drive such SCSTVDNs to achieve IES. By exploiting Lyapunov method and Kirchhoffs matrix tree theorem in graph theory, two types of sufficient IES criteria are respectively established in the form of Lyapunov-type theorem and coefficients-type theorem. Furthermore, to show the practicability of the theoretical results, the analysis of IES for the second-order Kuramoto oscillators with stochastic perturbations and time-varying delay is performed. A numerical example is finally provided to demonstrate the validity and feasibility of our analytical results.

1. Introduction

Over the past decades, coupled systems on networks (CSNs) have gained noticeable attention from food webs to ecological communities webs, communication networks to social organizations and the World Wide Web to the Internet [1–3]. The exploration of the dynamic behaviors of CSNs has also been a common task of an unprecedented array of basic subjects including physics, biology, sociology and engineering [4–7]. Therein, synchronization, a typical and interesting collective behavior of dynamics, has stirred much research interest owing to its fruitful applications in various fields [8–10]. Recently, most of previous studies are focused on synchronization among corresponding nodes between two or more networks that is referred to as outer synchronization [11,12]. Additionally, synchronization of all dynamical nodes within a network, which is called inner synchronization, also occupies a pivotal and vital position in applied science. For example, in [13], the authors showed how to use the inner synchronization technique to recognize an image, with strong robustness in recognition. Also in [14], the authors proposed an architecture of coupled networks to store and retrieve complex oscillatory patterns as inner synchronization states. Hence, studying the inner synchronization problem of CSNs is an important step for both understanding many natural phenomena and designing CSNs for practical use.

In practical implementation, time delay is ubiquitous since there naturally exists a consequence of finite information transmission and processing speeds among the units in many large-scale networks. As shown in [15,16], the existence of time delay could result in oscillatory behavior or instability. Furthermore, it has been observed that delay is usually variable as time. Thus, it is necessary to consider the effect of time-varying delay on inner synchronization of CSNs. On the other hand, CSNs are also perturbed by stochastic interferences which exist in external environment. In [17], stochastic disturbances are introduced, but the coupling between nodes is linear and internal delay is unconcerned. From our perspective, it will be

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better to take internal time-varying delay and random fluctuations into account in the models of CSNs for more realistic situation. Especially, it is critical to investigate the inner synchronization problem of stochastic coupled systems with time-varying delay on networks (SCSTVDNs). In [18], the synchronization problem for a class of continuous-time delayed complex networks with multiple stochastic disturbances was studied. The authors in [19] considered synchronization of delayed complex dynamical networks with impulsive and stochastic effects. Other results related to the issue were also obtained in [20–22].

In most of the cases, it is difficult to reach synchronization by itself for a network. In order to force it to realize inner synchronization, various effective control strategies have been developed, such as feedback control [23], adaptive control [24], impulsive control [25] and periodically intermittent control [26]. Among these approaches, periodically intermittent control, a discontinuous feedback control, has received increasingly researchers' attention since it can be used for a variety of purposes such as manufacturing, transportation and communication [27,28]. It is widely recognized that periodically intermittent control has a nonzero control width and can be easily implemented in practice. For example, in the process of signal transmission, when the signal becomes weak because of diffusion at the terminal, the external control can be loaded to achieve the desired result or requirement. And then, in order to reduce the control cost and the amount of transmitted information, the external control can be removed when the signal strength reaches the upper bound. Evidently, periodically intermittent control is economical and has good application value. Owing to those merits, periodically intermittent control has been applied to synchronize coupled networks, see [29], and the references therein. The exponential synchronization of neural networks under periodically intermittent control is studied in [30,31]. [32] discussed the cluster synchronization for directed networks with periodically intermittent control and [33] investigated the cluster synchronization of delayed complex networks via periodically intermittent control.

Up to now, various different types of methods are consecutively emerged to enrich the research topic of synchronization in the existing works. For instance, by utilizing the Lyapunov stability theory and linear matrix inequalities, Wang et al. [34] considered exponential synchronization of stochastic perturbed complex networks with time-varying delays. Other synchronization techniques have also imported, including $\mathcal{M}$-matrix approach [35]. As is shown in [36], coupled networks with or without perturbation can be expressed as a directed graph. So, a natural question arises: can we use the graph theory to study inner exponential synchronization (IES) of SCSTVDNs under periodically intermittent control? A positive answer is provided to the question and it is described in detail in this paper.

Li et al. [36,37] have done the pioneering work about the graph theory-based method to study the globally stability problem for coupled systems. Following in Li's footstep, plenty of researchers devoted themselves to the issue of CSNs in virtue of this approach and then some novel and useful results were gained in [38–42]. The authors in [39] discussed stability in probability for discrete-time stochastic coupled systems on networks by using the graph-theoretic approach. In [42], the graph-theoretic method was applied to study synchronization of stochastic coupled systems. It should be pointed that most of these researches concentrated on stability, boundedness and outer synchronization of coupled networks, and there are rare results about IES of coupled networks by the graph-theoretic method. Hence, inspired by the above analysis, the main purpose of this paper is to study the IES problem of SCSTVDNs under periodically intermittent control by combining the graph theory with Lyapunov method.

Compared with the existing results on the analysis of IES, the main contributions of this paper can be summarized as follows. Firstly, periodically intermittent controller together with time-varying delays and stochastic factor are all considered in the system. Secondly, by employing some results in graph theory, a Lyapunov-type theorem ensuring IES of SCSTVDNs under periodically intermittent control is established. What is more, a coefficients-type theorem is also derived, which can be readily checked in practice. Thirdly, to show the practicability of the theoretical results, the obtained criteria are applied to investigate IES of the second-order Kuramoto oscillators. In addition, a numerical example is provided to confirm the effectiveness of the main results.

The remainder of this paper is organized as follows. In Section 2, our mathematical model of the general SCSTVDNs is presented and some preliminary results are briefly outlined. The main results of this paper are arranged in Section 3. Therein, some sufficient criteria guaranteeing IES of SCSTVDNs are derived. Then Section 4 is devoted to the IES analysis of the second-order Kuramoto oscillators with stochastic disturbances and time-varying delay via periodically intermittent control. In Section 5, a detailed numerical example is given to show the validity of the obtained results.

Notations: The following notations will be used throughout this paper. Let $\mathbb{R}$ be the set of all real numbers, $\mathbb{R}^+$ be the set of all nonnegative real numbers and $\mathbb{R}^m$ be the m-dimensional Euclidean space. Define $\mathcal{N} = \{1, 2, \dots, n\}$ and $\mathbb{Z}^+ = \{1, 2, \dots\}$. The superscript "T" stands for the transpose. In addition, denote the family of all nonnegative functions which are continuously differentiable twice in x and once in t by $C^{2,1}(\mathbb{R}^m \times \mathbb{R}^+; \mathbb{R}^+)$. For $\tau > 0$, $C([-\tau, 0]; \mathbb{R}^{m(n-1)})$ is the family of continuous function x from $[-\tau, 0]$ to $\mathbb{R}^{m(n-1)}$ with norm $\|x\| = \sup_{-\tau \le s \le 0}|x(s)|$, where $|\cdot|$ is the Euclidean norm in $\mathbb{R}^m$. Let $(\bar{\Omega}, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\ge0}$ satisfying the usual conditions and $L^2_{\mathcal{F}_0}([-\tau, 0], \mathbb{R}^{m(n-1)})$ stand for the family of $\mathcal{F}_0$-measurable $C([-\tau, 0], \mathbb{R}^{m(n-1)})$-value random variables x such that $\mathbb{E}\|x\|^2 < \infty$. The mathematical expectation operator with respect to the given probability measure $\mathbb{P}$ is denoted by $\mathbb{E}\{\cdot\}$. Denote $B(t)$ as a one-dimensional Brownian motion defined on the probability space. The Laplacian matrix of $(\mathcal{G}, A)$ is defined as $L = (p_{jk})_{n\times n}$, where $p_{jk} = -a_{jk}$ for $j \ne k$ and $p_{jk} = \sum_{i\ne j} a_{ji}$ for $j = k$. Other basic concepts of graph theory involved in this paper can be referred to in [36,43].