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Consensus of multi-agent systems via hybrid impulsive protocols with time-delay

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ABSTRACT

This paper studies consensus problems of multi-agent systems. A novel hybrid consensus protocol with dynamically changing interaction topologies is designed to take the time-delay into account in both the continuous-time communication among agents and the instant information exchange at discrete-time moments. By using a new Halanay-type inequality and an auxiliary function, sufficient conditions are established to guarantee that the proposed consensus protocols lead to average-consensus. Our consensus criteria show that the networked multi-agent system with time-delay can achieve average-consensus with appropriate network topologies, suitably designed impulsive instants, and admissible time delays. Numerical simulations are provided to demonstrate the effectiveness of the theoretical results.

1. Introduction

A multi-agent system is a dynamic system consisting of a group of interacting agents distributed over a network. Consensus of a multi-agent system is an agreement problem among all the members of the multi-agent system. As one of the typical collective behavior, the consensus problem of multi-agent systems has attracted the attention of numerous researchers in recent years. This is mainly due to the widespread application of multi-agent systems in various areas, such as opinion formation, parallel computing, flocking, mobile robots, and distributed sensor networks (see, e.g., [1–5]).

Consensus protocol is a cooperative controller based on the local information of each agent over the network. In the past decades, many consensus protocols have been proposed to achieve consensus of multi-agent systems (see, e.g., [6–10]). Among them, the impulsive control method has been extensively applied to dynamical networks (see. e.g., [11–13]). Recently, various impulsive consensus protocols have been successfully applied to achieve consensus of multi-agent systems using only small impulses. On the other hand, it is practical to consider time-delay in processing the impulsive information (see, [14–16]). Up to now, most research on delayed impulsive consensus has been done on second-order multi-agent systems (see, e.g., [17–20]), and the obtained results are not applicable to protocol design problems for consensus of first-order multi-agent systems. For example, in [17], leader-following consensus of multi-agent systems has been studied with delayed impulsive protocols, but the delay in the impulse was required to be smaller than the length of each impulsive interval which was a rather restrictive condition on the impulse delay. To our best knowledge, very few work has been done on the consensus problem of multi-agent systems with delayed impulses. In [21], a hybrid impulsive consensus protocol was proposed to achieve the network consensus. However, the continuous-time and the discrete-time topologies were assumed to be the same, and also share the same communication delay size among agents. Then, in [22], an impulsive protocol was

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proposed with time-delay to study the consensus problems of multi-agent systems, and no continuous-time connections were considered in the network topologies. The hybrid consensus protocol has also been studied in [23] very recently, but time-delay was considered only in the impulsive part of the consensus protocol.

Motivated by the above discussion, we propose in this paper a novel hybrid impulsive consensus protocol with dynamically changing interaction topologies and different time delays in a continuous-time and discrete-time network. Although much research has been done on consensus problems of multi-agent systems with dynamically changing interaction topologies (see, e.g., [8,24–26]), this is to our best knowledge the first time to incorporate a hybrid impulsive consensus protocol with both dynamically changing topologies and time-delay. In this paper, all the network topologies are assumed to be balanced. However, only the discrete-time topologies are required to be strongly connected. Moreover, different topologies are considered in the continuous-time and discrete-time consensus protocols. A new Halanay-type inequality with delayed impulsive part is established, and then sufficient conditions on the relation among the delay size, the length of impulsive intervals, and the network fixed (or switching) topology are constructed by using the Halanay-type inequality and an auxiliary function. The upper bound for the admissible length of impulsive intervals is obtained. Uniform impulsive intervals are assumed throughout the paper to simplify the derivation of the main results, and illustrate the method of investigation of delays in the impulsive consensus protocols. However, it is straightforward to generalize our results to hybrid impulsive protocols with non-uniform impulses as studied in [21].

The rest of the paper is organized as follows. In Section 2, we shall formulate the consensus problem and propose the hybrid impulsive consensus protocol. The consensus results for multi-agent systems with fixed and switching topologies will be established, respectively, in Section 3. Two numerical examples will be provided to demonstrate the theoretical results in Section 4. The detailed proofs of the main results will be given in Section 5. Finally, in Section 6, conclusions will be stated.

2. Preliminaries

Throughout this paper, we denote by $\mathbb{R}$ the set of real numbers, $\mathbb{N}$ the set of positive integers, and $\mathcal{I}$ the set $\{1, 2, \ldots, n\}$. Next, we will introduce some basic concepts from graph theory and formulate the hybrid consensus problem.

2.1. Network topology

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a digraph (or directed graph) of order n with the set of nodes $\mathcal{V} = \{v_i \mid i \in \mathcal{I}\}$ and the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. An edge of $\mathcal{G}$ is denoted by (v_i, v_j) which means the node v_j can receive information from node v_i. The index set of neighbors of node v_i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} \mid (v_j, v_i) \in \mathcal{E}\}$. A directed path of digraph $\mathcal{G}$ is a sequence of edges $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), (v_{i_3}, v_{i_4}), \ldots$ in digraph $\mathcal{G}$. A digraph $\mathcal{G}$ is called strongly connected if there is a directed path connecting any two arbitrary nodes in $\mathcal{G}$.

A weighted digraph $\mathcal{G}_A = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ associated with a weighted adjacency matrix $\mathcal{A} = [\alpha_{ij}]_{n\times n}$ with nonnegative adjacency elements (or weighting factors) α_{ij} such that $(v_j, v_i) \in \mathcal{E}$ if and only if $\alpha_{ij} > 0$. It is assumed that $\alpha_{ii} = 0$ for all $i \in \mathcal{I}$. The graph Laplacian $\mathcal{L}$ of weighted digraph $\mathcal{G}_A$ is defined by $\mathcal{L} := \mathcal{D} - \mathcal{A}$ where $\mathcal{D} = diag\{d_1, d_2, \ldots, d_n\}$ with element $d_i := \sum_{v_j \in \mathcal{N}_i} \alpha_{ij}$ which is called the in-degree of node v_i. A weighted digraph $\mathcal{G}_A$ is said to be balanced if $\sum_{j=1, j\neq i}^{n} \alpha_{ij} = \sum_{j=1, j\neq i}^{n} \alpha_{ji}$ for all $i \in \mathcal{I}$.

2.2. Consensus protocols

Let $x_i \in \mathbb{R}$ denote the state of node v_i, and consider each node of a graph to be a dynamic agent with integrator dynamics

$$\dot{x}_i(t) = u_i, \quad i \in \mathcal{I}, \tag{1}$$

where u_i is a state feedback. We say that u_i is a protocol with topology $\mathcal{G}$ if the state feedback u_i only depends on the information of v_i and its neighbors, i.e., $u_i = u_i(x_{i_1}, x_{i_2}, \ldots, x_{i_j})$ and $v_{i_k} \in \{v_i\} \bigcup \mathcal{N}_i$ for $k = 1, 2, \ldots, j$. We say that a protocol u_i solves the average-consensus problem if and only if $\lim_{t\to\infty} \|x_i(t) - Ave(x(0))\| = 0$ for all $i \in \mathcal{I}$, where $Ave(x(0)) = \frac{1}{n}\sum_{j=1}^{n} x_j(0)$.

We consider the following consensus protocol based on a dynamically changing digraph $\mathcal{G}_A(t) = (\mathcal{V}, \mathcal{E}(t), \mathcal{A}(t))$ and a fixed digraph $\mathcal{G}_{\mathcal{A}'} = (\mathcal{V}, \mathcal{E}', \mathcal{A}')$:

$$\begin{aligned} u_i(t) = & \sum_{v_j \in \mathcal{N}_i(t)} \alpha_{ij}(t)[x_j(t - r(t)) - x_i(t - r(t))] \\ & + \sum_{k=1}^{\infty} \sum_{v_j \in \mathcal{N}'_i} \alpha'_{ij}[x_j(t - \tau_k) - x_i(t - \tau_k)]\delta(t - t_k), \end{aligned} \tag{2}$$

where r denotes the time-variant delay in the continuous-time consensus protocol satisfying $0 \le r(t) \le \bar{r}$ ($\bar{r}$ is a constant), and τ_k represents the time-delay in the discrete-time consensus protocol at time $t = t_k$ satisfying $0 \le \tau_k \le \bar{\tau}$ ($\bar{\tau}$ is a constant and $k \in \mathbb{N}$); $\alpha_{ij}(t)$ is the (i, j)th entry of the weighted adjacent matrix $\mathcal{A}(t)$ at time t, and $\mathcal{N}_i(t)$ denotes the set of node v_i's