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Practical output tracking control for switched nonlinear systems: A dynamic gain based approach

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ABSTRACT

This paper addresses the practical output tracking problem for a class of switched nonlinear systems with unstable subsystems. By using the single Lyapunov function (SLF) method and developing a dynamic gain based approach, the dynamic controllers of individual subsystems and a proper switching law are constructed to guarantee that all the signals of the closed-loop system are globally bounded while the tracking error between output and reference signal can be arbitrary small after a finite time. Two examples are given to demonstrate the effectiveness of the proposed control scheme.

1. Introduction

In recent years, global output tracking issue is one of the most important issues in control theory and engineering, and therefore has attracted considerable attention [1–4]. The works [1–3] included a fairly complete review and detailed report on the significant developments and achievements in the area of output regulation theory for linear and nonlinear systems. When the reference signal is prescribed, bounded and its time derivative is also bounded, the practical output tracking issue for nonlinear systems was studied in [4–8]. As a class of main hybrid systems, switched systems are composed of a family of continuous time subsystems and a switching rule. The inspiration for investigating switched systems derives from various environmental factors and intelligent control, such as aircraft control systems, robot control systems, and networked control systems [9–11]. However, the practical output tracking problem of switched systems has been limitedly studied in existing notes [12–14]. As a consequence, further investigation of practical output tracking for switched systems becomes much more important.

In this paper, we consider the practical output tracking problem for switched nonlinear systems of the form

$$\begin{aligned} \dot{x}_i &= g_{i\sigma(t)}(\bar{x}_i)x_{i+1}^{p_{i\sigma(t)}} + f_{i\sigma(t)}(x, u_{\sigma(t)}),\ i = 1, \dots, n-1, \\ \dot{x}_n &= g_{n\sigma(t)}(x)u_{\sigma(t)}^{p_{n\sigma(t)}} + f_{n\sigma(t)}(x, u_{\sigma(t)}), \\ y &= x_1, \end{aligned} \tag{1}$$

where $x = (x_1, \dots, x_n)^T \in \mathbb{R}^n$ and $y \in \mathbb{R}$ are the system state and output, respectively. For $i = 1, \dots, n$, $\bar{x}_i = (x_1, \dots, x_i)^T$. $\sigma(t)$ is a piecewise continuous (from the right) switching signal taking its values in a finite set $M = \{1, \dots, m\}$ and m is the number of subsystems. For $i = 1, \dots, n$, $k \in M$, $p_{ik} \in \mathbb{R}_{\geq 1} \triangleq \{\frac{p}{q} \geq 1 : p$ is a positive integer, and q is a positive odd integer$\}$, $u_k \in \mathbb{R}$ is the control input of the kth subsystem. $g_{ik} : \mathbb{R}^i \to \mathbb{R}$ and $f_{ik} : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$, are continuous functions for $i = 1, \dots, n$

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and $k \in M$. Moreover, we assume that the state of system (1) does not jump at the switching instants, i.e., the trajectory $x(t)$ is everywhere continuous. Let $y_r(t)$ be a $\mathcal{C}^1$ reference signal.

As is well-known, switched systems usually possess a complex behavior owing to the interaction between continuous dynamics and discrete dynamics. For instance, unconstrained switching may destabilize a switched system even if all subsystems are stable. On the contrary, constrained switching may stabilize a switched system even if all subsystems are unstable [15]. Therefore, Lyapunov stability and variations for individual subsystems become particularly important. Several methods, for example, common Lyapunov function (CLF), SLF, and multiple Lyapunov functions, have been proposed to handle the problem of stability and stabilization for switched systems [16–24]. As a matter of fact, when a CLF is found, the switched system is stable under arbitrary switchings. But the CLF approach is usually very hard to employ since such a CLF may be very difficult to find. Without a CLF, stability properties of switched systems often depend on switching signals. Thus an appropriate choice of a switching law plays a vital role in implementing stability of switched systems. In particular, the SLF method has been proved to be an effective tool for addressing stability problem of switched systems in [15,24,25]. The work [24] addressed global stabilization problem for a class of switched nonlinear systems in p-normal form, where the subsystems are not assumed to be asymptotically stable. The work [25] further investigated the practical output tracking problem of switched nonlinear systems in p-normal form, where the practical output tracking for all subsystems is not required. Subsequently, the work [26] studied the finite-time output tracking problem for a class of switched nonlinear systems in p-normal form by the convex combination method. To the best of our knowledge, the above control methodologies have conservativeness, which is generated by the derivative of virtual controllers in the iterative procedure. Immediately, an interesting question is raised: *Under weaker assumptions, how can we design controllers of subsystems and a proper switching law to achieve practical output tracking of switched nonlinear systems via a less conservative control method?*

Motivated by the method in [27,28], we present a new design method to deal with the practical output tracking problem for switched uncertain nonlinear system (1) by adding a power integrator technique. Compared with the previous literatures on switched or non-switched control, the prominent features of this paper are summarized as follows:

(i) Since the virtual controller directly counteracts coupling terms and nonlinearities, the design method in [24,25] has explicit conservativeness (see Remark 4.1 for details). Therefore, a new design method combining the SLF method and a dynamic gain based approach is proposed to decrease conservativeness of control method in this paper.

(ii) By a dynamic gain based approach, the previous works [27,28] focus on global stabilization problem for non-switched nonlinear systems. But the practical output tracking problem is not solvable. Thus we extend above control method to address practical output tracking issue for switched nonlinear systems with unstable subsystems.

(iii) To simplify the form of controllers for individual systems, a series of dynamic updating laws are delicately designed.

(iv) Under weak condition that the power orders are allowed to be ratios of positive integers over odd integers, the dynamic state-feedback controllers of subsystems and a proper switching law are designed by constructing a SLF.

This paper is organized as follows. Preliminaries are included in Section 2. Section 3 exhibits the main result which consists of controller design and stability analysis. In Section 4, two illustrative examples are given to demonstrate the effectiveness of the proposed control method. Conclusions are provided in Section 5.

Notations Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional real space; $\mathbb{R}_+$ represents the set of all the nonnegative real numbers; For a given vector or matrix X, X^T is its transpose; $|\cdot|$ interprets the Euclidean norm; $\mathcal{C}^i$ denotes the set of all functions with continuous ith partial derivatives. Let $\bar{\xi}_i = (\xi_1, \ldots, \xi_i)$ and $\bar{l}_i = (l_1, \ldots, l_i)$, $i = 1, \ldots, n$.

2. Preliminaries

To solve practical output tracking problem for switched nonlinear system (1), we provide some useful lemmas which will be used in later controller design.

Lemma 2.1 ([5]). *For real numbers $a \geq 0$, $b > 0$ and $m \geq 1$, the following inequality holds:*

$$a \leq b + \left(\frac{a}{m}\right)^m \left(\frac{m-1}{b}\right)^{m-1}.$$

Lemma 2.2 ([29]). *For positive real numbers p, q, there is a positive real-valued function $\rho(x, y)$ such that*

$$|x|^p|y|^q \leq \frac{p}{p+q}\rho(x,y)|x|^{p+q} + \frac{q}{p+q}\rho(x,y)^{-p/q}|y|^{p+q}, \ x \in \mathbb{R}, \ y \in \mathbb{R}.$$

Lemma 2.3 ([29]). *For $x \in \mathbb{R}$, $y \in \mathbb{R}$, and the constant $p \geq 1$, the following inequalities hold:*

$$|x+y|^p \leq 2^{p-1}|x^p + y^p|, \ \ (|x| + |y|)^{\frac{1}{p}} \leq |x|^{\frac{1}{p}} + |y|^{\frac{1}{p}} \leq 2^{\frac{p-1}{p}}(|x| + |y|)^{\frac{1}{p}}.$$

If $1 \leq p \in \mathbb{R}_{odd}$, *then*

$$|x^{\frac{1}{p}} - y^{\frac{1}{p}}| \leq 2^{1-\frac{1}{p}}|x-y|^{\frac{1}{p}}, \ \ |x^p - y^p| \leq p|x-y|(x^{p-1} + y^{p-1}) \leq c|x-y||(x-y)^{p-1} + y^{p-1}|$$

with c being a positive constant.