Nonlinear Analysis: Hybrid Systems 30 (2018) 189–198

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Distributed consensus of the nonlinear second-order multi-agent systems via mixed intermittent protocol

Wanli Guo *, Haijun Xiao

College of Mathematics and Physics, China University of Geosciences, Wuhan 430074, PR China

A R T I C L E   I N F O

Article history:
Received 7 October 2017
Accepted 25 May 2018

Keywords:
Leader–follower consensus
Second-order multi-agent system
Intermittent control
Adaptive control

A B S T R A C T

The leader-following consensus problem for second-order multi-agent systems is investigated in this paper. The topology of the system is directed and the dynamics of each agent is nonlinear. Based on the intermittent control strategy and the adaptive control method, a mixed protocol for each follower is designed. Using the Lyapunov stability theory, sufficient conditions are derived such that all the followers can track the leader. Finally, some numerical simulation results are presented to demonstrate the effectiveness of the proposed consensus protocol.

1. Introduction

Consensus is an essential task of cooperative control for multi-agent systems, which means that all agents' states converge to the same vector under a distributed protocol. Due to its numerous applications in real practice, consensus problem for the multi-agent systems has attracted increasing attention from different disciplines [1–5].

Leader-following consensus is a special case of consensus where the task is for all the followers to track the leader asymptotically. The leader-following coordination of multi-agent systems has many potential applications. For example, to dodge the hazardous obstacles, several autonomous vehicles (viewed as leaders) equipped with necessary sensors can guide the other vehicles (viewed as followers) into a specific area safely [6]. Therefore, there are numerous results reported on the leader-following consensus of the multi-agent system in the past few years. In light of agent dynamics, the consensus problem of multi-agent systems is mainly divided into two categories: the first-order consensus problem and the second-order consensus. In contrast to the first-order consensus problem, the second-order consensus has obtained more attention. For example, in Olfati-saber [7], the flocking algorithms are proposed under the assumptions that all the agents should connect to the virtual leader and also the virtual leader travels at a constant velocity. Su et al. [8] extends the results in Olfati-saber [7] from two aspects, i.e., the case that the virtual leader travels at a constant velocity and the case that the virtual leader travels at a time-varying velocity. Distributed observers [9–11] were designed to estimate such position and velocity information when the position and velocity information of the virtual leader are not available to the followers in real time. In [12], the consensus tracking problem was investigated for the second-order multi-agent system with a self-active leader. Aiming at the lag consensus of second-order nonlinear multi-agent systems, a control protocol for each follower based on local information of neighboring agents was proposed, and an adaptive feedback control protocol was also given in [13]. In [14], the authors investigated the robust consensus tracking problem for a class of heterogeneous second-order nonlinear multi-agent systems with bounded external disturbances. In [15], the leader-following consensus problem for second-order multi-agent systems with updated control gains was studied. In [16], necessary and sufficient conditions were

* Corresponding author.
E-mail address: guowanliff@163.com (W. Guo).

https://doi.org/10.1016/j.nahs.2018.05.004
1751-570X/

derived for reaching second-order leader-following consensus by combining the algebraic graph theory and the analytical method. The group consensus problem of second-order nonlinear multi-agent systems through leader-following approach and pinning control was addressed in [17].

It has been observed that most of the works mentioned above studied the consensus problem in multi-agent systems with the assumption that the information is transmitted continuously among the agents. However, this may not be the case in reality due to temporary sonar equipment failures or the existence of communication obstacles and so on. That is to say, in some cases, the agents can only communicate with their neighbors on some disconnected time intervals. To deal with this problem, the intermittent consensus protocol has been proposed in some works. In [18], the authors investigated second-order consensus problem for multi-agent systems with nonlinear dynamic and directed topologies, where each agent can only communicate with its neighbors on some disconnected time intervals. In [19], a novel adaptive intermittent control protocol was first introduced, where only intermittent relative local information was used. In order to stabilize the whole network, the required length of the intermittent control intervals was derived by using directed graph theory and Lyapunov stability analysis in [20]. By using intermittent control, the issue of consensus for leader-following multi-agent systems was concerned in [21]. In [22], the cluster synchronization for linearly coupled networks with constant time delay was investigated by pinning periodically intermittent controllers. In [23], without assuming that the mobile agents can communicate with their neighbors all the time, the consensus problem of multi-agent systems with general linear node dynamics and a fixed directed topology was investigated. In [24], under the common assumption that the multiple agents can communicate with their neighbors only during a sequence of discontinuous time intervals, a new neighbor-based consensus control protocol based on observers was developed for each agent.

On the other hand, adaptive consensus strategies which can appropriately tune the strength with the dynamic evolution of the network have been proposed in many current works. In [25], for the case where the directed graph was strongly connected, a distributed adaptive protocol was designed to achieve consensus. In [26], distributed adaptive protocols and Lipschitz distributed adaptive protocols were respectively designed for consensus problems of the leader-following multi-agent systems. To reaching the consensus, a novel adaptive protocol was proposed based only on the relative state information in [27]. In [28], an adaptive estimation scheme was designed by virtue of the relative position measurement and the relative velocity measurements from its neighbors for the bipartite consensus of a competition networks. A distributed intermittent communication framework was proposed via adaptive approach to consider the robust adaptive consensus tracking for higher-order multi-agent uncertain systems with nonlinear dynamics in [29]. The focus of this reference was the problem of actuator with occasional failure inputs and communication resources constraints. Similarly, in [30], the protocol based on the adaptive intermittent control ideas was designed to concern second-order consensus problem of nonlinear multi-agent systems with mixed time-delays and intermittent communications.

Motivated by the above analysis, this paper addresses the leader-following consensus problem for the second-order multi-agent systems. Many works focus on agents described by linear dynamics. However, in reality, the agents may be governed by inherent nonlinear dynamics and hence the consensus problem with nonlinear dynamics is more challenging. So the dynamics of each agent considered in this paper is nonlinear. Based on the intermittent control strategy and the adaptive control method, a mixed protocol for each follower is designed in which the followers' information can be shared directly between each other, but the leader's information is obtained adaptively. Using the Lyapunov stability theory, sufficient conditions are derived such that all the followers can track the leader. Finally, some numerical simulation results are presented to demonstrate the effectiveness of the proposed consensus protocol.

The rest of this paper is organized as follows. The problem is formulated and some necessary definitions, lemmas, and assumptions are given in Section 2. Main results are discussed in Section 3. Examples and their simulations are obtained in Section 4. Finally, the paper is concluded in Section 5.

Throughout this paper, the following notations will be used. The norm of a vector $u \in R^n$ is defined as $\|u\| = (u^T u)^{1/2}$; $1_n = (1, 1, \dots, 1)^T \in R^n$; $\lambda_{\max}(A)$ denotes the maximal eigenvalue of the matrix $A \in R^{n\times n}$; $\mathrm{diag}(d_1, d_2, \dots, d_n)$ denotes a diagonal matrix with diagonal elements being $d_1, d_2, \dots, d_n$; for real symmetric matrix P, $P > 0$ ($P \geq 0$) means that P is positive (semi-) definite; $A \otimes B$ denotes the Kronecker product of matrices A and B.

2. Preliminaries and problem formulation

Let $G = (V, \varepsilon, A)$ be a weighted directed graph of order n with the set of vertices $V = \{1, 2, \dots, n\}$, the set of directed edges $\varepsilon \subset V \times V = \{(i, j) : i, j \in V\}$, and a weighted adjacency matrix $A = (a_{ij})_{n\times n}$, where $a_{ii} = 0$ and $a_{ij} \geq 0$ for $i \neq j$. $a_{ij} > 0$ if and only if $(j, i) \in \varepsilon$ which means that the information flow goes from vertex j to vertex i. If $(j, i) \in \varepsilon$, the vertex j is called a neighbor of i. The set of neighbors of vertex i is denoted by $N(i) = \{j \in V : (j, i) \in \varepsilon, j \neq i\}$. A path between vertex i and vertex j in G is a sequence of edges $(i, i_1), (i_1, i_2), \cdots, (i_l, j)$ in the graph with distinct vertices $i, i_1, i_2, \dots, i_l, j$. If there exists a special vertex that has a directed path to all the other vertices, then G is said to have a spanning tree. This special vertex is called the root. A digraph is strongly connected if for any pair of vertices i and j, there is a directed path from vertex i to vertex j.

The out-degree matrix of G is $D = \mathrm{diag}(d_1, \dots, d_n)$, where diagonal elements $d_i = \sum_{j=1}^{n} a_{ij}$ for $i = 1, 2, \dots, n$. Then the Laplacian of the weighted digraph G is defined as $L = D - A$ which is a zero row sum and non-positive off-diagonal elements matrix.

In what follows, we are mainly concerned with a graph $\bar{G}$ consisting of n followers (related to graph G) and a leader. To describe the digraph $\bar{G}$ completely, we define $B = \mathrm{diag}(b_1, \dots, b_n)$ as the leader adjacency matrix associated with $\bar{G}$, where