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Exponential stability for generalized stochastic impulsive functional differential equations with delayed impulses and Markovian switching[☆]

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ABSTRACT

This paper is concerned with exponential stability for a class of generalized stochastic impulsive functional differential equations with delayed impulses and Markovian switching. A novel subsequence approach of the impulsive and switching time sequence is introduced to cope with the impulsive control problem with large and small delays. Based on the stochastic Lyapunov function and Razumikhin technique, a dwell time bound and related criteria are established to ensure the pth moment exponential stability, almost surely exponential stability and uniform stability of the trivial solutions. The main advantage of the proposed algorithm lies in that the delay bound and parameters are not necessarily required, which are commonly used to restrict the dwell-time bound and the decay rate of Lyapunov function. Finally, two examples are performed to demonstrate the usefulness of the main results.

1. Introduction

Markovian switching systems are a particular class of hybrid systems that have been extensively studied in the past decades [1,2]. Typically, a Markovian switching system possesses several operation modes and the system modes switching is governed by a Markov process. Markovian switching systems can be used to model many physical systems undergoing random abrupt changes in their structure and hence find numerous applications in practice, such as manufacturing systems, aircraft control, target tracking, solar receiver control and power systems [3–5].

Impulsive systems are often used to describe the dynamics of processes that are subject to abrupt changes at discrete moments [6,7]. In the last few decades, impulsive control has been considered as a powerful tool in the stability analysis of nonlinear dynamical systems [8–10]. Time delay is encountered in practice and often causes poor system transient response even instability of the systems [11,12]. It should be pointed out that delays are only assumed appearing in continuous dynamics [13–15]. In fact, in transmitting the impulse information, input delays often happen. For example, the network output information is delivered via a digital communication network [16,17]. Obviously, computation time and network induced delays make it possible that the kth input update time reaching the destination may be greater than the kth sampling time. This case may cause delay in the discrete impulse dynamics. So, it is essential to consider the effects of delayed impulses since they may have destabilizing effects.

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On the other hand, stochastic functional differential equation has increasingly attracted great interests in both theoretical research and practical applications [18–20]. Although these models are important for many complex processes, they do not cover those phenomena displaying certain kinds of dynamics with impulses. A dynamical system with stochastic and impulsive effects can be adequately described as stochastic impulsive system or stochastic impulsive differential equations [21]. During the past decades, many asymptotic stability and exponential stability results concerning stochastic impulsive systems have been derived [22–33]. For example, in [13,14,34], the exponential stability problems of impulsive differential systems with delay are respectively studied based on Lyapunov–Razumikhin method and Lyapunov–Krasovskii technique. Cheng and Deng [22] establish some sufficient exponential stability conditions in which the bound of impulses and the decay rate of Lyapunov function are estimated for a class of impulsive stochastic differential systems. By using Lyapunov method, the pth moment and almost sure exponential stability for impulsive stochastic functional differential equations with finite delay are reported in [23]. In [35], the criteria of pth moment asymptotic stability are obtained for stochastic differential systems with Markovian switching and some restrictions are imposed on the decay rate of Lyapunov function. Further, Wu et al. [36] obtain some less conservative results which loose the constraints in [35]. Though the stability of stochastic impulsive differential equations has stirred some initial research interest, there still leaves much room for reducing the possible conservativeness. In [36], the pth moment stability and pth moment exponential stability are investigated, which seems difficult to be applied to deal with the globally exponential stability. In [13,14,22,24], the dwell time approach is utilized and it is required to satisfy constraints involving certain delay bounds and a parameter. However, these conditions are difficultly adapted to other general systems. In [27–29], some requirements are imposed on the growth of impulse, i.e., ($d_k + e_k \leq 1/\gamma$, in Theorem 3.1 in [27], $e^{c\delta} < 1/(\rho_1 + \rho_2) < q$ in [28], and $e^{2c\alpha} \leq q$ in Theorem 1 in [29]), which restricts the application of those results. It is noted that the results in [13–15,29,30] may not be applicable to large delays directly. This is probably because the relationship between the decay of system trajectories and the delay size as well as the dwell-time bound is overlooked. The aforementioned discussion has aroused an look into the following questions: (Q1) What technique can be adopted to address the obtained stability criteria for both cases of larger and smaller delays than the dwell-time bound? (Q2) Under what conditions imposed on the impulses and the dwell-time bound, the results will be less conservative?

In this paper, we are motivated to deal with the exponential stability problem for a family of generalized stochastic impulsive functional differential equations with delayed impulses and Markovian switching. Several novel sufficient conditions are obtained to guarantee the exponential stability of the systems in this paper. It should be pointed out that all the mentioned conclusions in the above literatures do not consider the delay, delayed impulse, stochastic interferences and Markov switching simultaneously. The main contributions are highlighted as follows: (1) a subsequence of the switching and impulsive time sequence is constructed to prove the pth moment exponential stability of system. The stability condition obtained in this paper only require the Lyapunov function to be nonincreasing along each time interval, of the specially organized subsequences of he "switching and impulsive time". This subsequences design method allows us to handle both cases that the bound of delay is smaller or larger than the dwell-time bound; (2) compared with [19,22,24], we need no delay bound nor parameter (such as α in Liu [24]) to limit the dwell-time bound. Moreover compared with the existing literatures, more general impulsive systems are taken into consideration and thus better reflect the reality.

2. Preliminaries

R, Z and Z^+ denote the sets of real, nonnegative integer and positive numbers, respectively. I denotes the identity matrix with compatible dimensions. $\omega(t) = (\omega_1(t), \omega_2(t), \ldots, \omega_m(t))^T$ is an m-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, P)$ with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$. Given $\tau > 0$, let $PC([-\tau, 0]; R^n)$ denote the family of piecewise right continuous functions ϕ from $[-\tau, 0]$ to R^n with the uniform norm defined by $\|\phi\|_\tau = \sup_{-\tau\leq\theta\leq 0} |\phi(\theta)|$. Denote by $\mathcal{L}^p_{\mathcal{F}_0}([-\tau, 0]; R^n)$ the family of all $\mathcal{F}_0$ measurable, $PC([-\tau, 0]; R^n)$-valued stochastic variables $\phi = \{\phi(\theta) : -\tau \leq \theta \leq 0\}$ such that $\sup_{-\tau\leq\theta\leq 0} E|\phi(\theta)|^p < \infty$, where E stands for the corresponding expectation operator with respect to the given probability measure P.

$\{r(t), t \geq 0\}$ denotes a right-continuous Markov chain on a complete probability space $(\Omega, \mathcal{F}, P)$ taking values in a finite state space $S = \{1, 2, \ldots, N\}$ with generator $Q = (q_{ij})_{N\times N}$ given by

$$P\{r(t + \triangle t) = j|r(t) = i\} = \begin{cases} q_{ij}\triangle t + o(\triangle t) & if\ i \neq j \\ 1 + q_{ii}\triangle t + o(\triangle t) & if\ i = j, \end{cases}$$

where $\triangle t > 0$ and $\lim_{\triangle t\to 0} o(\triangle t)/\triangle t = 0$. Here, $q_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$ while $q_{ii} = -\sum_{j\neq i} q_{ij}$. We assume that the Markov chain $r(\cdot)$ is independent of the Brownian motion $\omega(\cdot)$.

Similar to Refs. [37–39], we suppose that t_k, $k \in Z^+$ is a given monotonically increasing sequence, $t_1 < t_2 < \cdots < t_k < \cdots$. It is known that almost every sample path of $r(t)$ is a right-continuous step function with a finite number of simple jumps in any finite subinterval of R. So, $r(t)$ is a constant in every interval $[t_{k-1}, t_k)$ for any $k \geq 1$, i.e.,

$$r(t) = r(t_{k-1}), \quad \forall t \in [t_{k-1}, t_k), \ k \geq 1.$$