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Stationary distribution and ergodicity of a stochastic hybrid competition model with Lévy jumps

Meng Liu *, Yu Zhu

School of Mathematical Science, Huaiyin Normal University, Huaian 223300, PR China

A R T I C L E I N F O

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Taking white noises, Markovian switching and Lévy jumps into account, an $n-$species Lotka–Volterra competitive model with random perturbations is proposed and studied. Sufficient conditions for the existence and uniqueness of an ergodic stationary distribution of the model are established. These sufficient conditions are sharp in some cases. Some interesting results are revealed: the white noises and Lévy jumps could make the stationary distribution disappear as well as appear; the Markovian switching could make the stationary distribution appear. Some numerical simulations are also introduced to illustrate the main results.

1. Introduction

In the natural world, it is a common phenomena that several species compete for the limited resources. Moreover, the growth of the species is often affected by random perturbations [1,2]. Hence it is of great significance to investigate the multi-species competitive models with random perturbations. A classical multi-species competitive model with random perturbations is the following Lotka–Volterra stochastic competitive model perturbed by white noises:

$$\mathrm{d}N_i(t) = N_i(t)\left(b_i - \sum_{j=1}^{n} h_{ij}N_j(t)\right)\mathrm{d}t + \alpha_i N_i(t)\mathrm{d}W_i(t),\ \ i = 1, \ldots, n, \tag{1}$$

where $N_i(t)$ represents the population size of the ith species at the time t; $b_i > 0$ is the growth rate of the ith species; $h_{ii} > 0$ denotes the intraspecific competition rate while $h_{ij} > 0\ (i \neq j)$ stands for the interspecific competition rate, $i, j = 1, \ldots, n$; $\{W(t)\}_{t\geq 0} = \{W_1(t), \ldots, W_n(t)\}_{t\geq 0}$ is an n-dimensional Brownian motion defined on a complete probability space $(\Omega, \{\mathcal{F}_t\}_{t\geq 0}, P)$, α_i^2 stands for the intensity of the stochastic noise. Owning to its theoretical and practical significance, model (1) and its various generalizations have received great attention. For example, Jiang et al. [3] considered the persistence in time average, extinction and the existence of an ergodic stationary distribution of model (1); Nguyen and Yin [4] and Liu and Fan [5] considered permanence and extinction of model (1) with $n = 2$; Li and Mao [6] studied permanence, extinction and global attractivity of model (1) in the non-autonomous case; Liu et al. [7] carried out the survival analysis of model (1) in a polluted environment; Tan et al. [8] analyzed model (1) with impulses; Model (1) with time delays was explored in [9–12].

White noises cannot describe some sudden environmental perturbations (for example harvesting, epidemics, earthquakes) which are often encountered in the growth of species. Bao et al. [13] used a Lévy jump process to describe these

* Corresponding author.
E-mail address: liumeng@hytc.edu.cn (M. Liu).

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phenomena initially, and proposed the following stochastic competitive model with jumps:

$$\mathrm{d}N_i(t) = N_i(t^-)\left\{\left(b_i - \sum_{j=1}^{n} h_{ij}N_j(t^-)\right)\mathrm{d}t + \alpha_i \mathrm{d}W_i(t) + \int_{\mathcal{U}} \lambda_i(u)\tilde{\Upsilon}(\mathrm{d}t, \mathrm{d}u)\right\},\ i = 1, \ldots, n, \tag{2}$$

where $N_i(t^-)$ stands for the left limit of $N_i(t)$, $\mathcal{U} \subseteq (0, +\infty)$, $\tilde{\Upsilon}(\mathrm{d}t, \mathrm{d}u) = \Upsilon(\mathrm{d}t, \mathrm{d}u) - \theta(\mathrm{d}u)\mathrm{d}t$, Υ is a Poisson counting measure with characteristic measure θ satisfying $\theta(\mathcal{U}) < +\infty$. Under the assumption that

$$1 + \lambda_i(u) > 0,\ u \in \mathcal{U},\ i = 1, \ldots, n. \tag{3}$$

Bao et al. [13] investigated the existence, uniqueness, boundedness, the sample Lyapunov exponent and extinction of the solutions of model (2). Especially, for one-dimensional model (3) (i.e., Logistic case), Bao et al. [13] established the explicit solution of the model and gave the sufficient conditions under which the model has a unique stationary distribution. See also [14–17] among others for some recent results on population models with Lévy jumps.

It has been noted that [18] the growth rates of species are also often influenced by the telephone noises, for example, the growth rates of some species in dry season are much different from those in rainy season [19–21]. Telegraph noise can be regarded as a switching between some regimes of environment [19–21]. Some scholars [19–25] have pointed out that the telephone noises can be described by a continuous-time-finite-state Markov chain. Following this, in this paper, we consider the following stochastic regime-switching competitive model with jumps

$$\mathrm{d}N_i(t) = N_i(t^-)\left\{\left(b_i(\beta(t)) - \sum_{j=1}^{n} h_{ij}N_j(t^-)\right)\mathrm{d}t + \alpha_i(\beta(t))\mathrm{d}W_i(t) + \int_{\mathcal{U}} \lambda_i(u)\tilde{\Upsilon}(\mathrm{d}t, \mathrm{d}u)\right\},\ i = 1, \ldots, n, \tag{4}$$

where $\beta(t)$ is a continuous-time Markov chain with finite-state space $\mathbb{S} = \{1, 2, \ldots, m\}$.

In the investigation of population models, positive equilibrium state and its stability is one of the most interesting and important topics. However, when random perturbations are considered, most stochastic population models do not keep the positive equilibrium state that the original deterministic population models own. Therefore, in the investigation of stochastic population models, the stability of the "stochastic positive equilibrium state"—the existence of the stationary distribution becomes one of the most interesting topics [22,26–30]. However, to the best of our knowledge, there are no results relative to the existence of stationary distribution of model (4) that have been reported, and the effects of white noises, Lévy jumps and Markovian switching on the existence of stationary distribution of model (4) are still unknown.

Motivated by these, in this paper we consider model (4) and establish the sufficient conditions for the existence and uniqueness of an ergodic stationary distribution in Section 2. Some recent results on special cases of model (4) are improved greatly. We also show that these sufficient conditions are sharp in some cases. In Section 3, we introduce some numerical simulations to illustrate the effects of white noises, Lévy jumps and Markovian switching on the existence of stationary distribution of model (4), and give some concluding remarks.

2. Main results

In this paper, as a standing hypothesis we always suppose that Υ, $\beta(t)$ and $\{W(t)\}_{t\geq 0}$ are independent. We also suppose that $\beta(t)$ is irreducible. Hence the Markov chain $\beta(t)$ is ergodic and has a unique stationary distribution $\pi = (\pi_1, \ldots, \pi_m)$. For the sake of convenience, we introduce some notations:

$$\mathbb{R}^n_+ = \{x \in \mathbb{R}^n | x_i > 0,\ i = 1, \ldots, n\},\ H = \det((h_{ij})_{n\times n}),$$

$$a_i(k) = b_i(k) - 0.5\alpha_i^2(k) - \int_{\mathcal{U}}\Big[\lambda_i(u) - \ln(1 + \lambda_i(u))\Big]\theta(\mathrm{d}u),\ \bar{a}_i = \sum_{k=1}^{m} \pi_k a_i(k),$$

$$v_i(k) = a_i(k) - \sum_{j=1, j\neq i}^{n} \frac{h_{ij}}{h_{jj}} a_j(k),\ \bar{v}_i = \sum_{k=1}^{m} \pi_k v_i(k),\ m_i(t) = \int_0^t \int_{\mathcal{U}} \ln(1 + \lambda_i(u))\tilde{\Upsilon}(\mathrm{d}s, \mathrm{d}u),\ i = 1, \ldots, n.$$

Denote by H_i the determinant obtained by replacing the ith column of H with $(\bar{a}_1, \ldots, \bar{a}_n)^{\mathrm{T}}$. According to Lemma 4.1 in Golpalsamy [31] (Page 294), if $\bar{a}_i > 0$ and $\bar{v}_i > 0$, $i = 1, \ldots, n$, then the equations $(h_{ij})_{n\times n}X = \bar{a}$ have a unique positive solution $\left(\frac{H_1}{H}, \ldots, \frac{H_n}{H}\right)^{\mathrm{T}}$, where $X = (x_1, \ldots, x_n)^{\mathrm{T}}$, $\bar{a} = (\bar{a}_1, \ldots, \bar{a}_n)$. We also need the following two assumptions

Assumption 1. There exists a positive constant $\hat{K}$ such that

$$\int_{\mathcal{U}}\Big[\ln(1 + \lambda_i(u))\Big]^2\theta(\mathrm{d}u) < \hat{K},\ i = 1, \ldots, n, \tag{5}$$

$$\int_{\mathcal{U}} \lambda_i^2(u)\theta(\mathrm{d}u) < \hat{K},\quad i = 1, \ldots, n. \tag{6}$$

That is to say, the intensity of Lévy noise is not too large.