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An integrated production, inventory and preventive maintenance model for a multi-product production system

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ABSTRACT

This paper considers a production system that can produce multiple products alternately. Products go through the system in a sequence and a complete run of all products forms a production cycle. An integrated production, inventory and preventive maintenance model is constructed, which is characterized by the delay-time concept. Two different situations are studied based on whether the unqualified products and downtime caused by the failures of the system, set-up and preventive maintenance can be ignored or not. Three cases are considered for each situation, depending on the position of the preventive maintenance epochs: the first case, where preventive maintenance is carried out at the end of each production cycle; the second case, where preventive maintenance is carried out at each set-up time of the products; and the third case, where preventive maintenance is carried out at some set-up times only, since it may not always be optimal to carry out preventive maintenance at the end of the production cycle or at each set-up time. The modeling objectives are to find the optimal number of production cycles per year and the optimal position of preventive maintenance that will maximize the expected profit per unit time. Numerical examples, using real data, are presented to illustrate the model.

1. Introduction

Researchers have studied production lot-sizing problems extensively for many years [1]. The economic benefit of a production system could be improved by dividing the demand of the products into several smaller lot sizes to reduce the inventory cost, but this must be balanced with the increased set-up cost. The classical economic production quantity (EPQ) model calculates the optimal lot size of the product by minimizing the sum of the inventory holding and set-up costs [2,3]. Two facts are often ignored in production lot-sizing researches: first, the production system can fail during the course of production, so preventive maintenance (PM) should be considered in conjunction with lot-size determination; second, the classical EPQ model only considers a single type of product going through the production system, but in reality, in typical batch production situations, multiple products are often produced on the same production system. In such a case, the lot sizes of different products are related and the production system will not be idle when a product lot size is completed.

There are some related researches, though they are not directly applicable to the case we describe in this paper. For instance, Porteus [4] introduced a relationship between the product quality and the lot size by assuming that the production process goes 'out-of-control' with a given probability each time it produces an item. He concluded that producing smaller lot sizes could result in a smaller fraction of defective units. Rosenblatt and Lee [5] derived a similar conclusion by assuming an exponential process shift distribution. Lee and Rung [6] studied lot-sizing policies in multi-stage serial production systems with the systems prone to failures. They concluded that the lot sizes in the unreliable systems could be smaller or larger than those in the classical EPQ model. Sami [7] considered a system that deteriorates with an increasing failure rate, and proposed a model to determine the optimal number of production runs and the PM schedule that minimizes the long-term average cost. Chakraborty et al. [8] developed integrated production, inventory and maintenance models to study the joint effects of process deterioration, machine breakdown and inspections on the optimal lot-sizing decisions. For some other relevant works, see [9–12]. These works are more reasonable than the classical EPQ model as the system failures and maintenance are considered; however, all these models are restricted to the one-product case.

In the real world, to fully utilize the production system, several products are usually produced by the same system alternately. For example, in a steel factory, different types of steel products are produced in the same production system alternately to meet the variety of customers' demands. If the demands of all products are

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fixed, the demands are typically divided into smaller lot sizes and go in sequence in the production system. Whenever a product lot size is completed, the production system needs to be set up again in order to take a different product. This set-up time can be utilized as a PM window so that the PM causes less interruption to the regular production process and thus saves cost. However, at which set-up time to carry out the PM is a decision problem not well studied in existing literature.

In this paper, we consider the problem of jointly determining the optimal lot sizes and PM policy for a production system that can produce multiple products alternately. We assume that the demand for each product is fixed, and the total supply per year for each product is well balanced at the production planning stage based on the demand and the capacity of the production system. This implies that the quantities to be produced for all products per year are fixed and match the production capacity. However, they can be produced in one big lot size or several smaller lot sizes. Our task is to determine the optimal lot size for each product in conjunction with the PM policy. Intuitively, smaller lot sizes will lead to smaller inventory costs, more opportunities for PM but more set-up costs, and vice versa.

Our research into this problem was actually motivated from our recent observation in a factory that produces cast iron pipes for the construction industry. Six different sizes of cast iron pipes are alternately produced by the same centrifugal system. The aggregate annual demands of all pipes are fixed, and they are divided into smaller lot sizes to be produced to meet the individual demands of the customers and reduce the inventory costs. When a lot size is completed, the system needs to be set up again where the pipe mold – which is a part of the centrifugal system – needs to be changed in order to produce a differently sized pipe. PM is usually carried out at some set-up time epochs, but also at the time when all production lots are completed. We believe that this situation can also be observed from other batch production systems.

We start with a simple case where k products ($k=2,3,4,\ldots$) are produced by the same production system with two different PM policies: either carrying out the PM after completing all product lot sizes, or at each set-up time of the products. Then, for the general case, carrying out PM at some set-up times and not at others, we propose a profit model in which the optimal PM interval is decided first by ignoring the set-up point, and then by searching for which set-up is closer to this interval by comparing the distance. Using the same PM interval, and from this set-up point, we continue the above process until the end of the production cycle. Finally, we can get the optimal PM policy based on this approach.

To model the PM policy for the system, we use the delay-time concept. The concept of the delay-time has been widely applied in maintenance modeling and optimization; see [13] for a detailed review. The failure process of a component is regarded as a two-stage process: the period from new to the initial point of the defect, usually referred to as the normal stage or time-to-defect; and the period from the initial point to the component failure, termed as the delay-time stage [13], as shown in Fig. 1. If the PM, which is considered to be perfect, is carried out during the delay-time, the defect can be removed by repair or replacement. Delay-time models have been applied to single-component systems [14,15], and complex systems with many components [16–20]. In this paper, we use the complex system delay-time model since typical production systems are equipped with many components. For complex systems, multiple defects can be present at any one time, and the arrival process of the defects can be approximated by a Homogeneous Poisson Process (HPP) [13]. Fig. 2 shows an example of the defect arrival and failure processes of a complex system if PM interventions were carried out at points A and B. From Fig. 2, it is clear that three defects could be identified and removed if the defect identification and removal are perfect. The delay-time-based PM models differ from other PM models in that they directly model the relationship between the PM and the number of system failures. Many case studies have shown the validity of the delay-time-based models: see [21–23].

This paper is organized as follows. The problem description, model assumptions and notations are given in Section 2. Section 3 introduces the profit models based on different PM policies, assuming that the system downtime caused by failures, set-ups and PMs can be ignored and the system is free from producing unqualified products. Section 4 incorporates the consideration of unqualified products and system downtime into the profit models. Numerical examples with real data are presented in Section 5. Section 6 concludes the paper.

2. Problem description, assumptions and notations

2.1. Problem description

We consider a production system that can produce multiple products but is subject to PM at some set-up times. First, we introduce the concept of a production cycle. A production cycle is a complete run of all products. Specifically, we restrict our attention to the case where each product is produced exactly once within a production cycle and the cycle is repeated over time. Regarding the issue of repeated production, it is true that some products may have small demands and may be produced in only one or several cycles, and then not repeated again. This particular scenario will be studied in a separate paper, since the production cycle will be uneven and different modeling techniques are required. We restrict our attention to the case where a product is produced only once in a cycle because (as shown below) producing a product more than once in a cycle will lead to potential idle time or production shortage.

To demonstrate that producing a product more than once within a cycle is not ideal, consider the case of three products. We assume that in one production cycle there are two productions for product 1, and one production for each of product 2 and product 3. We use T_i ($i=1,2,3$) to denote the production time for one lot of the ith product. Fig. 3(a) shows the ideal situation, and the production schedule is perfect. The production time of product 2 must be equal to the production time of product 3 ($T_2=T_3$), otherwise idle or shortage may occur, as shown in Fig. 3(b) and (c). It is clear that this arrangement is impractical and becomes worse if more products are produced. Therefore, we focus on the situation that all products are produced once in a production cycle.

2.2. Assumptions

(1) The demands of all products are fixed and can be divided into smaller lot sizes that are produced according to a fixed sequence, which is preset.

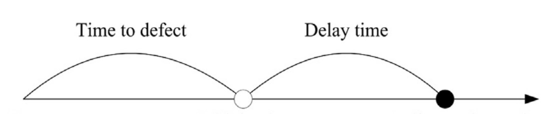

Fig. 1. The delay-time for a defect. '○' initial point, '●' failure point.

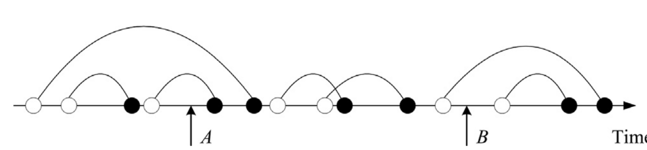

Fig. 2. The defect arrival and failure processes of a complex system.