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Robust stability of switched positive linear systems with interval uncertainties via multiple time-varying linear copositive Lyapunov functions

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ABSTRACT

This paper studies the robust stability analysis of a class of switched positive linear systems with uncertainties in the framework of dwell time switching. The uncertainties refer to interval uncertainties. Two classes of dwell time switching signals are considered in this paper: (i) the first class is confined by a certain pair of upper and lower bounds; (ii) the other is the minimum dwell time. First, a class of multiple time-varying linear copositive Lyapunov functions is constructed to analyze the robust stability of the studied switched system. Then, under the pre-given dwell time switching, the sufficient conditions are obtained by restricting the decay of the Lyapunov functions of the active subsystem and forcing "energy" of the overall switched system to decrease at switching instants by the proposed Lyapunov functions. All present conditions are solvable in terms of linear programming. An example is considered in order to emphasize the effectiveness of the proposed approach.

1. Introduction

Switched systems are one the important classes of hybrid systems, which contains of a family of subsystems and a rule orchestrating the switching between them [1]. In the real world, switched systems have been widely used in various control engineering, such as power electronics, chemical processes, power systems, mechanical systems, and so on [2–11]. The main concern in the study of switched systems is the issue of stability, which has been intensively studied in the literature [1,12,13]. How to analysis stability such an appropriate switching became one of the most challenging problems in the study of switched systems [14,15]. The existence of a common Lyapunov function for all subsystems was shown to be a necessary and sufficient condition for a switched system to be asymptotically stable under arbitrary switchings [1,16,17]. Most switched systems in practice, however, do not possess a common Lyapunov function, yet they still may be asymptotically stable under some properly chosen switching. There are mainly three main switching laws: state-dependent switchings [18], and dwell-time-based methods [19], and state-dependent dwell-time [20].

On the other hand, switched positive systems have attracted considerable attention over several decades [21–24] since such switched systems can be used to model many practical systems, e.g., economics, biology, sociology, and communication. A number of conditions have been given towards the stability under arbitrary switchings for switched positive linear systems by a common vector-parameterized copositive Lyapunov function [25], a common quadratic copositive Lyapunov

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function [26], a switched linear copositive Lyapunov function [27], and so on. On the other hand, when the switched systems are not stable under arbitrary switchings, it is interested in determining, if possible, switching strategies that ensure the convergence to zero of the state trajectories [28–31]. It is known that a dwell time of active subsystem can subside possible large state transients [32–34]. In addition, the dwell time technique appears to be a good alternative for studying electrical circuits with physical switches or cases with sudden component faults in electrical and mechanical systems. Therefore, this paper will choose dwell time technique to study the switched positive systems.

Uncertainties are frequently encountered in practical systems [35,36], such as communication systems, chemical systems, and transportation systems. It is well known that even small uncertainties may decline the system performance or even destroy stabilities, and also make difficulties for the stability analysis of systems. It is therefore important to take into account the presence of uncertainties in the practical analysis of systems. For switched systems (not positive) with uncertainties [37,38], there have been many available results on the interval uncertainties [39], polytopic uncertainties [40], and external disturbance input [41]. However, there are a few results on switched positive systems with uncertainties, even though this class of systems has extensive applications. The main reason lies in that it is a more challenging work. One needs to consider not only the stability but also the positivity when dealing with the problems of switched positive systems, which is interesting but complex. [42] investigated the stabilization of continuous- and discrete-time positive switched systems with uncertainties by multiple linear copositive Lyapunov functions associated with linear programming, respectively. It is worth noting that there is till a lot of room for improvements in the aforementioned results since there exist some restrictions. For example, the average dwell time is obtained in [42] by computing some linear programming, but given in advance. Multiple *time-varying* linear copositive Lyapunov functions may reduce some conservatism. Thus, questions naturally arise: is it possible to study the robust stability problem under any pre-given dwell time switching signal? If possible, under what conditions can we achieve this goal and how? To our best knowledge, in the literature there have few results which provide answers to these questions, which is the motivation of the present paper.

From the motivation above, this paper investigates the robust stability of a class of switched linear positive systems with uncertainties in the two classes of the dwell time switching signals: (i) the first class is confined by a certain pair of upper and lower bounds; (ii) the other is the minimum dwell time. Only the interval uncertainties are considered in this paper. A class of multiple time-varying linear copositive Lyapunov functions is first constructed, and then the sufficient conditions in terms of linear programming are obtained by restricting the decay of the Lyapunov function of the active subsystem and forcing "energy" of the overall switched system to decrease at switching instants by the proposed Lyapunov function to guarantee the robust stability of the studied switched systems with the pre-given dwell time switching signal. A numerical example is considered in order to emphasize the effectiveness of the proposed approach. The main contributions of this manuscript are summarized as follows: (1) A type of multiple time-varying linear copositive Lyapunov functions candidate is constructed to the robust exponentially stability analysis, which can be reduced to the class of time invariant linear copositive Lyapunov functions used in [42]. The functions in our paper may be exploited to relax conservativeness. (2) Unlike [42] where a feasible solution to linear programming is first computed and then the dwell time is obtained, in our paper a dwell time is pre-given and then for such a dwell time check if there exists a feasible solution to linear programming. (3) By carefully choosing parameters, the conditions on the robust exponentially stability in our result can be reduced to the ones in [42].

The organization of the paper is as follows. Section 2 presents the Preliminaries and systems description. Main results are given in Section 3. An illustrative example is presented to demonstrate the effectiveness of the proposed method in Section 4. Finally, some conclusions are drawn in Section 5.

Notations: N is the set of nonnegative and positive integer, $\Re$, $\Re^n$, and $\Re^{n\times n}$ are the sets of real numbers, n-tuples of real numbers, and the space of $n\times n$ real matrices, respectively. $\|\|$ is the Euclidean norm. A^T is the transpose of matrix A. I_n is the $n\times n$ identity matrix. a_{ij} stands for the element in the ith row and the jth column of A, and $A\succ 0$ ($A\succeq 0$) means $a_{ij}>0$ ($a_{ij}\geq 0$) for $i,j=1,2,\ldots,n$. $A\succ B$ ($A\succeq B$) means $a_{ij}>b_{ij}$ ($a_{ij}\geq b_{ij}$) for $i,j=1,2,\ldots,n$. $\underline{\rho}(v)$ and $\overline{\rho}(v)$ stand for the maximal and minimal element of vector v, respectively.

2. Preliminaries and systems description

In this paper, we consider the following class of switched positive linear systems with interval uncertainties:

$$\dot{x}=A_{\sigma(t)}x,\ \ x(t_0)=x_0, \tag{1}$$

where $x(t)\in\Re^n$ is the system state, $x_0\in\Re^n_+$ is a constant vector, $\sigma(t):[0,\infty)\to S=\{1,2,\ldots,M\}$ is the switching signal, which is assumed to be a piecewise constant or piecewise continuous (from the right) function depending on time, $M\geq 2$ is the number of models (called subsystems) of the switched system. For any $p\in S$, $A_p\in\Re^{n\times n}$ is unknown constant matrices. When $t\in[t_k,t_{k+1})$, $k\in N$, we call the $\sigma(t_k)$th subsystem is active. We assume that the state of the switched system (1) does not jump at the switching instants, i.e., the trajectory $x(t)$ is everywhere continuous.

Our goal is to identify certain classes of switching signals to guarantee the robust exponential stability of the switched system (1) under the following assumption.

Assumption 1 (*[42]*)**.** For each A_p in system (1), there are known a Metzler matrix $\underline{A}_p$ and a Hurwitz matrix $\overline{A}_p$, such that $A_p\in\left(\underline{A}_p,\overline{A}_p\right)$, where $\underline{A}_p=\left[\underline{a}_{ij}^{(p)}\right]$, $A_P=\left[a_{ij}^{(p)}\right]$, and $\overline{A}_P=\left[\overline{a}_{ij}^{(p)}\right]$.