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Hybrid output regulation for nonlinear systems: Steady-state vs receding horizon formulation

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ABSTRACT

The hybrid output regulation problem in the presence of periodic jumps is approached in this paper for a class of nonlinear hybrid systems satisfying rather mild assumptions. We provide sufficient conditions that characterize steady-state trajectories achieving output regulation that are defined, mimicking the linear case, in terms of two equations: the first one describes the solution of a *flow-only* output regulation problem, while the second is associated to an auxiliary output regulation problem concerning the *monodromy* equivalent system of the *flow zero-dynamics*. Such conditions are then revisited and a *receding-horizon*, *steady-state-less*, solution to the (hybrid) output regulation problem is suggested, based on the solution of a sequence of two-point boundary value problems.

1. Introduction

Output regulation and tracking problems consist in enforcing a response of the controlled system in such a way that a given output of interest of the plant evolves over time following a desired profile generated by a reference system, regardless of external disturbances which are typically only known to belong to certain classes of signals [1–3]. Output regulation represents, together with stabilization, one of the fundamental problems in control theory, and as such has been an active research area for more than four decades [2–5]. Compared to stabilization, however, the fascinating feature of output regulation consists in the fact that complex steady-state evolutions may be considered, as induced by the *exogenous* signals. Departing from the basic formulation provided in the linear time-invariant case [2,1,3], several research directions have been pursued, culminating with the definition of the output regulation problem in the nonlinear context [4], see also the more recent monographs [6,7] for a more complete survey.

More recently, motivated by the ubiquitous interaction between digital devices and continuous processes, the increasing attention devoted to the hybrid framework (see [8] for a comprehensive introduction) has encouraged the study of the output regulation problem for classes of hybrid linear and nonlinear systems. Clearly, since these systems are described by *flow* (continuous-time) dynamics as well as *jump* (discrete-time) dynamics, this problem becomes significantly more involved than the purely continuous-time (or purely discrete-time) counterpart. Hence, even in the linear setting, very few contributions are available in the literature.

Output regulation for completely general classes of hybrid systems is a largely unexplored territory, due to a number of difficulties making the general hybrid regulation problem much more intricate then the classic (non-hybrid) one. However, a framework in which hybrid regulation can be addressed for *hybrid systems* with some sort of continuity with respect to the classic case has been singled out in [9]: this consists in focusing on a class of hybrid systems for which jump times are *a priori* fixed as multiples of a basic value, similarly to the classic single rate sampled-data systems. Such restriction on

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the jump times avoids the above mentioned problem, meanwhile preserving most of the interesting behavior due to the interplay between flow and jump dynamics that has to be carefully taken care of in order to achieve regulation. The same class of systems has been further investigated by the same authors *e.g.* in [10], with partial extensions to the nonlinear case in [11]. The linear, multi-input multi-output case (possibly with more inputs than outputs) has been addressed by [12–14], removing all assumptions of minimum phaseness or on the relative degree.

In this paper, mostly focusing on the preliminary structural problem of identifying motions compatible with zero regulation error, a twofold contribution is given. First, sufficient conditions of output regulation for a class of *nonlinear* hybrid systems in the presence of periodic jumps are provided, extending the conditions provided in [15,16] for the *linear* setting. In this context, it is shown that, in the nonlinear case as well, the purely continuous-time contribution to the hybrid solution, referred to as the *heart of the hybrid regulator*, has a crucial role, just like the presence of *redundant inputs* (namely, the fact that the plant has a number of inputs *strictly larger* than the number of outputs, which is not required in non-hybrid output regulation).

The main differences with respect to the previous works [12,13,15,17,16,14] can be summarized as follows. Firstly, the case of *nonlinear plant and exosystem* is considered, whereas previous works were limited to the linear case. It is remarked that the techniques developed in [15] heavily rely upon the explicit characterization of solutions to the underlying (linear) ordinary differential equations, whose use is questionable in the nonlinear setting. Technically, dealing with the nonlinear case implies, among other things, that the monodromy dynamics can be expressed in terms of flows and jumps in a format reminiscent of the linear case, although it cannot be explicitly computed, and thus requires alternative approaches in order to arrive at explicitly computable solutions. Hence, as a second contribution, a rather different and unconventional path is pursued, and the problem of defining a steady-state response achieving output regulation is sidestepped by proposing to achieve output regulation via a *receding horizon* approach, *without* the need of defining a steady-state. Such an approach allows to circumvent the above mentioned difficulties in computing the flow in closed form, and apparently has never been used or studied even in the linear context. The problems solved in a receding horizon fashion are actually two-point boundary value (ode) problems, which can also be enriched with additional (state or input) constraints as well as performance index to be optimized. Note, in fact, that the receding horizon solution, involving the computation of the control input separately on each flow interval, opens the door to the possibility of achieving special features which were not even considered in previous approaches, even in the linear case. As an example, it would be possible to optimize different cost functions on different flow intervals, possibly choosing the cost of interest in real-time during operation; such a feature is especially desirable in applications, since it gives the opportunity to achieve different higher level objectives, while preserving the achievement of output regulation. Finally, the implementation of the receding horizon solution is permitted by the interesting *geometric characterization* of the required conditions. Essentially, this point amounts to provide, and analyze in terms of suitable relations among subsets of the state space, the conditions under which an output regulation problem which involves the evolution on a complete (infinitely long) time domain can be decomposed and solved in a sequential fashion, simply by computing the solution on each single flow interval $[t_k, t_{k+1}]$ and disregarding other flow intervals, as far as suitable conditions are satisfied.

The rest of the paper is organized as follows. After establishing some preliminaries in Section 2, the aim of Section 3 consists in formalizing the definition of the output regulation problem under examination together with some basic notation and preliminary results. The derived sufficient conditions are then presented in the full information case in Section 4, in which similarities and (crucial) differences with respect to the linear case are discussed. In Section 5 an interesting interpretation of the hybrid output regulation problem is provided and the task is formulated in terms of a sequence of two-point boundary value problems, thus introducing a *receding-horizon* solution to output regulation. The underlying geometric picture is sketched in Section 6. The paper is concluded by numerical simulations on two academic examples in Section 7.

2. Notation and preliminaries

In this paper we focus on a special class of hybrid systems considered in [10] for the linear case, characterized by having all solutions defined on the same *hybrid time domain*[1]

$$\mathcal{T} := \{(t,k) : t \in [t_k, t_{k+1}],\, k \in \mathbb{N}\}, \qquad t_h := \begin{cases} 0, & \text{if } h = 0, \\ \varphi + (h-1)\tau_M, & \text{if } h \in \mathbb{N}_{\geq 1}, \end{cases} \tag{1}$$

with $\tau_M > 0$ given, where t denotes the current value of continuous time and k denotes the number of jumps already occurred. For a function $\chi(\cdot,\cdot)$ defined on $\mathcal{T}$, denote as usual $\dot{\chi}(t,k) := \frac{d}{dt}\chi(t,k)$ and $\chi^+(t,k) := \chi(t,k+1)$, provided that $t = t_k$. Solutions to the considered hybrid systems are piecewise absolutely continuous functions $\chi(\cdot,\cdot)$ that satisfy a differential equation (flow dynamics) $\dot{\chi} = f(\chi)$ almost everywhere in $\mathcal{T}$, and moreover they satisfy a difference equation (jump dynamics) $\chi^+ = g(\chi)$ when $(t,k) \in \mathcal{T}$ is such that $t = t_k$, $k \in \mathbb{N}_{\geq 1}$. Note that, while for general hybrid systems [8] it is necessary to explicitly define the *flow set* (in which the solution is allowed to flow) and the *jump set* (in which the solution is allowed to jump), having the fixed time domain (1) implies that there is no ambiguity about when the solution is flowing and when it is

[1] An equivalent way to introduce such time domains (see [8,10]) consists in introducing *clocks* described by the hybrid dynamics $\dot{\tau} = 1$ for $\tau \in [0, \tau_M]$ and $\tau^+ = 0$ for $\tau = \{\tau_M\}$ by choosing of $\tau(0,0) = \varphi$. Since such clocks are not exploited in this paper, a relevant notational simplification is achieved by using the equivalent, explicit definition (1).