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Hybrid robust discrete sliding mode control for generalized continuous chaotic systems subject to external disturbances

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ABSTRACT

This paper presents a discrete sliding mode control (DSMC) for the robust chaos suppression of the generalized continuous-time chaotic systems subject to matched/mismatched disturbances. The proposed DSMC first ensures the existence of the sliding manifold. Then, the effect of external disturbances including matched and mismatched cases are discussed after the controlled system is driven into the sliding manifold. The proposed results show that the chaotic behaviour of controlled systems with matched disturbances can be fully suppressed to zero or robustly driven into an estimated bound in the state–space form, which was not explicitly addressed in the literature. Finally, two different chaotic systems are utilized to study the hybrid chaos suppression. The corresponding numerical results show the effectiveness and robustness of the proposed DSMC.

1. Introduction

Chaos is a kind of special characteristics existing in nonlinear systems, which is a bounded unstable dynamic behaviour that exhibits sensitivity depending on the initial conditions and contains infinite unstable periodic motions in the strange attractor. Although the noise-like behaviour of chaos might be favourable to system design, such as chaotic secure communication [1–5], the chaos behaviour in most engineering systems is highly undesirable because it will affect the performance or damage the mechanical structure of systems and needs to be suppressed. Therefore, the study of nonlinear control techniques for chaos suppression has emerged as a new and attractive field. As a result, several feedback control methods leading to suppression or synchronization of chaos have been proposed in the literature, such as sliding mode control [6–11], passive control [12], impulsive control [13,14] and OGY method [15]. However, these approaches only developed for continuous controlled systems. An alternative to the afore-mentioned feedback control methods is the non-feedback control methodology, which usually utilizes the given external or parametric excitations to the nonlinear plant to control the behaviour of the chaotic system [16,17]. Although there exist well-developed control theory and design methods available in the literature for designing an analogous controller to improve the performance, the designed analogous controller is often required to be implemented by a digital one for better reliability, lower cost, smaller size, more flexibility and better performance. Therefore, research on the discrete-time control for chaotic systems has become intensified in recent years [18–22]. Nevertheless, on chaos suppression, most studies mentioned above are only focused on some special types

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of chaotic systems without considering the external mismatched disturbances. In addition, the complex continuous-time controller is often implemented to achieve the desirable control goals.

As well known, robust control aims to asymptotically remove the effect of matched/mismatched disturbances. By surveying the existing robust control methods [22–24] for both continuous and discrete systems, to fully remove the effect of bounded plant uncertainties or external disturbances, they need to satisfy the matched condition or depend on the system state. Recently, many studies for systems with mismatched disturbances can be founded in [25–28]. The influence of mismatched disturbances can be suppressed in the sense of H_∞ control [25] but cannot be fully removed. In [26], based on a finite time disturbance observer, a nonsingular terminal sliding mode control approach is proposed for mismatched disturbance attenuation. In [27], a continuous dynamic sliding-mode control with a sliding-mode differentiator is proposed for both high-order matched and mismatched disturbances. In [28], by designing a sliding surface based on the disturbance estimation, the system states can be driven to the desired equilibrium asymptotically even in the presence of mismatched disturbance. Although, the proposed methods in [26–28] can well solve the problems of mismatched disturbance attenuation. They are only for the systems with a single input $u(t) \in R$. Furthermore, their controllers are continuous and cannot be directly implemented by a digital microprocessor controller for better reliability and performance. During the past three decades, sliding mode control has been emerged as a distinctive robust control strategy for many kinds of engineer systems [6–11,22–28]. Depending on the proposed switching surface and sliding mode controller, the trajectories of dynamic systems are able to be guided to the fixed sliding manifold. In addition, the SMC approaches proposed above have two main features which are the reducing order of dynamics from the purposed switching functions and the robustness of the restraining system with matched disturbances. However, it should be mentioned that the property of reducing order in conventional sliding mode control makes it difficult to deal with the stability analysis of the systems with mismatched disturbances.

For the above reasons, in this paper, we study the discrete sliding control design for the generalized continuous-time chaotic systems with matched/mismatched disturbances. A novel DSMC scheme is proposed to solve the chaos suppression problem, which can be effectively applied to all general classes of chaotic systems. A special type of switching surface function is proposed to avoid the reducing order of systems, which makes easy to estimate the stability of the closed-loop system in sliding manifold for both matched and mismatched disturbances. The proposed results ensure that the chaotic behaviour of controlled systems with matched disturbances can be fully suppressed to zero or robustly driven into an estimated bound in the state–space form. Also the every state of controlled systems with mismatched disturbances can be individually evaluated the control performance, which was not explicitly addressed in the literature. Finally, two illustrative examples are used to demonstrate the effectiveness and robustness of the proposed design method.

Note that throughout the remainder of this paper, the notation M^T is used to denote the transpose for a square matrix M, while for $x \in R^n$, $\|x\| = \left(x^T x\right)^{1/2}$ denotes the Euclidean norm of the vector. $\|A\| = \left[\lambda_{\max}(A^T A)\right]^{1/2}$ is the matrix norm of A. I_n is the identity matrix of $n \times n$. $Sign(S) = [sign(s_1), sign(s_2) \ldots, sign(s_m)]^T \in R^m$ and $sign(s)$ is the sign function of s, if $s > 0$, $sign(s) = 1$; if $s = 0$, $sign(s) = 0$; if $s < 0$, $sign(s) = -1$. $a \geq b$ means $a_i \geq b_i$ for vectors $a = \begin{bmatrix} a_1 & a_2 & \ldots & a_n \end{bmatrix}^T$ and $b = \begin{bmatrix} b_1 & b_2 & \ldots & b_n \end{bmatrix}^T$.

2. System description and DSMC design

In this paper, we consider the hybrid chaos control of the generalized continuous-time chaotic systems. A DSMC is utilized to control the generalized continuous-time chaotic systems. To complete the hybrid control design, in what follows the discretizing model of an analogue system is necessarily determined. Consider the generalized continuous chaotic systems described by

$$\dot{x}(t) = Ax(t) + Bg(x, t), \tag{1}$$

where $x(t) \in R^n$ is the state vector of the system, $g(x, t) \in R^m$ is the nonlinear vector, $A \in R^{n\times n}$ and $B \in R^{n\times m}$ are the system matrices. To generally describe the chaotic systems in the true physical world, the controlled nonlinear system (1) is assumed to be subjected to unknown external disturbances and can be rewritten as follows

$$\dot{x}(t) = Ax(t) + B(g(x, t) + u(t)) + B_d d(t), \tag{2}$$

where $u(t) \in R^m$ is the control input vector introduced to suppress the chaos behaviour of systems, $d(t) \in R^r$ is the unknown external disturbance but satisfies $\|d(t)\| \leq \gamma$, $B_d \in R^{n\times r}$ is the system disturbance matrix. The augmented system of (2) can be represented as

$$\dot{x}(t) = Ax(t) + \overline{BU}(t), \tag{3}$$

where $\overline{B} = \begin{bmatrix} B & B_d \end{bmatrix} \in R^{n\times(m+r)}$ and $\overline{U}(t) = \begin{bmatrix} g^T(x, t) + u^T(t) & d^T(t) \end{bmatrix}^T$. Then the discretization of system (3) is given as

$$x(kT + T) = Gx(kT) + \overline{HU}(kT), \tag{4}$$

where T is the sampling period and $G = e^{AT}$; $\overline{H} = [G - I_n]A^{-1}\overline{B}$ [29]. It is noticed that when A is a singular matrix, the matrix $\overline{H} = [G - I_n]A^{-1}\overline{B}$ becomes $\overline{H} = T\sum_{i=0}^{i=\infty}[(AT)^i/(i+1)!]\overline{B}$.