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Synchronization criteria for singular complex networks with Markovian jump and time-varying delays via pinning control[☆]

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This paper makes a study on the problem of synchronization for singular complex networks with Markovian jumping parameters and mixed time-delays. The system consists of N nodes which switch from one mode to another according to a Markovian chain with known transition probability. Based on the strategies of pinning control, the singular complex networks are synchronized. A new type of criterion for synchronization can be obtained by means of utilizing the appropriate Lyapunov–Krasovskii function, the linear matrix inequality (LMI) approach, stochastic analysis techniques and the convexity of matrix functions. Finally, simulation examples are employed to demonstrate the effectiveness of the proposed method.

1. Introduction

During the past few years, great attention has been paid to the study of complex network systems due to their ubiquity in the natural world, so does the phenomenon of synchronization. There are some significant interests in the study of synchronization from different fields, for instance, secure communication, modeling brain activity and pattern recognition phenomenon [1–5].

As is well-known to all, Markov jump system, a special class of hybrid system, is specifically made up of two components. The first component refers to the mode described by a continuous-time finite-state in the process of Markov. While, the second one refers to the state which is represented by a system of differential equations. The application of the Markovian jump systems is respectively mentioned in networks, control systems, manufacturing systems, economic systems as well as modeling production systems to some extent. The analysis of stability on the system of Markovian jump neural networks can be found in [6–11]. On the other hand, the systems of singular Markovian jump with mode-dependent and singular matrices have attracted a broad concern in articles [12–14], which consist of variety of physical processes, such as, power systems and circuit systems. These systems are called generalized systems, differential–algebraic systems or implicit systems. It has been noted that considerable results of regular (non-singular) systems have been extended to network systems [15–19]. Although there are a few articles about singular complex network system [20–23], the Markov process is not taken into consideration a lot. In this paper, not only the system of singular complex networks is considered, but also the stochastic process is considered, in which the article has a certain universality.

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Time-delays are presented in many physical processes due to the length of time. In another words, a long-time period can be attributed to its occurrence. It is known clearly that time-delays can destabilize the behavior of networks. Thus, a system with time-delays may be complicated and interesting. There are a lot of papers [24–35] involved in time-delays. Therefore, the problem of complex networks with time-delays is an attractive topic, just as the articles [25,26] stated. These papers have discussed synchronization stability of continuous/discrete complex networks with interval time-varying delays. It should be noticed that time-delays occurred in these papers are assumed to be varying in an interval where the lower and upper boundaries are clearly known. The pattern of discrete-time complex dynamical networks is discussed in [27]. And the mode-dependent mixed time-delays are presented in [30]. In terms of an interval siding mode controller, projective lag synchronization of Markovian jumping neural networks has been studied. The problems of synchronization of T–S fuzzy complex dynamical networks with time-delays, impulsive delays and stochastic effects are studied in [32] according to T–S fuzzy methods and LMIs approach.

Theoretically speaking, control will be a necessary means used for guiding or forcing the network to achieve desired synchronization, which is fit for the condition that a given network of dynamical systems is not synchronized or the synchronized state is not an expected one. In practice, it may be impossible and unnecessary to control every node in a dynamical network with huge number of nodes. Therefore, a pinning control strategy shown in [36–50], can be employed to achieve the goal by directly adding control inputs to a fraction of nodes selected from the network, which is of great significance for the control of networks system. The author in this paper [38] investigated the cluster synchronization problem of a class of general complex dynamical networks under pinning control. The pinning controllers are designed according to the nodes. In article [41], it mainly studied the finite-time synchronization problem for a class of neutral complex dynamical networks with Markovian switching by utilizing the pinning control technique and constructing the appropriate stochastic Lyapunov–Krasovskii function. The adaptive pinning control in [42] has been investigated. Meanwhile, some other methods are also be used in order to achieve synchronization, for example, the sampled-data control, impulsive control, adaptive control and sliding mode control and so on. According to the author in [51], the complex dynamical networks are synchronized by means of using the sampled-data control. Pinning sample-data control in [52] for synchronization of complex networks has been studied with probabilistic time-varying delays via the approach of quadratic convex.

According to previous studies, the system of synchronization with Markovian jumping stochastic hybrid couplings and both mixed time-delays have not been explored broadly. It should be noted that the mixed time-delays are comprised of both the discrete and distributed delays under the Markovian jumping mode. In this paper, the author investigated the problem of synchronization for singular complex dynamical networks with Markovian jumping parameters via pinning control approach, built up a Lyapunov–Krasovskii function and applied the integral inequality technique into use in order to make a deep analysis so as for some less conservative results in terms of linear matrix inequalities (LMIs). The proposed LMIs can be easily solved by using Matlab LMI Toolbox. Considerable examples are presented in the following part to illustrate the effectiveness of the proposed results.

Notation: The standard notations will be used throughout this paper. R^n and $R^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. For real asymmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means $X - Y$ is semi-positive definite (respectively, positive definite). I is the identity matrix with appropriate dimension. The symbol "$*$" is used to represent a term that is induced by symmetry. The superscript T stands for the transpose of a matrix or a vector, $\mathrm{diag}(\cdots)$ denotes a block-diagonal matrix. Let $\tau > 0$ and $C([-\tau, 0], R^n)$ denote the family of continuous functions ϕ, from $[-\tau, 0]$ to R^n. Moreover, $(\Omega, F, \{F_t\}, P)$ is a complete probability space with a filtration, $\{F_t\}_{t\geq 0}$ satisfying the usual conditions. $E\{\cdot\}$ represents the mathematical expectation. $\lambda_{\max}(\cdot)$ means the largest eigenvalue of a matrix. If not explicitly stated matrices are assumed to have compatible dimensions.

2. Model description and mathematical preliminaries

The following section is mainly involved in the Markovian jumping singular complex networks with two additive mode-dependent time-varying delays consisting of N nodes, in which each node is an n-dimensional dynamical subsystem:

$$\begin{aligned} E\dot{x}_k(t) &= -D(r(t))x_k(t) + A(r(t))f(x_k(t)) + B(r(t))g(x_k(t-\tau_1(t))) \\ &\quad + C(r(t))\int_{t-\tau_2(t)}^{t} h(x_k(s))ds + c\sum_{j=1}^{N} G_{kj}\Gamma(r(t))x_j(t-\tau_1(t)) \end{aligned} \tag{1}$$

where $x_k(t) \in R^n$ is the state vector associated with n nodes. E is the singular matrix satisfying $rank(E) = r$ $(0 < r \leq n)$, $D(r(t)) = diag\left\{d_{1,r(t)}, d_{2,r(t)}, \ldots, d_{n,r(t)}\right\} > 0$; $A(r(t)), B(r(t)), C(r(t)) \in R^{n\times n}$ are respectively, the connection weight matrix, the discretely delayed connection weight matrix, and the distributively delayed connection weight matrix;$\tau_1(t)$ is the discrete time-varying delay satisfying $0 \leq \tau_1(t) \leq \tau_1, 0 \leq \dot{\tau}_1(t) \leq d_1$ for some positive scalars τ_1 and d_1. $\tau_2(t)$ is the distributed time-varying delay satisfying $0 \leq \tau_2(t) \leq \tau_2, 0 \leq \dot{\tau}_2(t) \leq d_2$ for some positive scalars τ_2 and d_2. $c > 0$ represents coupling strength; $\Gamma(r(t))$ is an inner-coupling matrix; $G = (G_{ij})_{N\times N}$ denotes the coupling configuration matrix, if there is a