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Stabilization of switched linear systems via admissible edge-dependent switching signals

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ABSTRACT

This paper proposes and studies the concept of admissible edge-dependent average dwell time (AED-ADT) which characterizes a kind of new switching signals for switched systems. The switching behavior of the proposed switching signal is denoted by a directed graph, where each admissible transition edge (ATE) signifies a switching from one subsystem to another. New stability criteria for switched systems with AED-ADT switching are then derived. The choices of transition weights of ATEs make the proposed AED-ADT switching more general than mode-dependent average dwell time (MDADT) switching. Stabilization conditions for switched linear systems are also established by designing state-feedback switching controller whose gain matrices can be obtained by solving linear matrix inequalities. The effectiveness of the proposed approaches is demonstrated by a numerical example.

1. Introduction

Stability analysis of switched linear systems has received considerable attention, and the switching mechanism is of greater importance to guarantee the stability of switched systems by dwell time approach [1]. These problems are motivated partly by physical or man-made systems whose working conditions change very rapidly from time to time, such as flight control systems [2], power electronics [3] and partial element equivalent circuits [4] etc. Investigations on stability of switched systems are generally classified into two categories. One is stability under arbitrary switching which is concerned with stability conditions of switched systems under all possible switching signals. Another is stability under constrained switching which considers system performances and the relationships between switching signals and system stability. In this paper, we concentrate on stability and stabilization of switched systems under constrained switching.

Early works on the stability under constrained switching can be traced back to about twenty years ago [1,5]. Arbitrarily fast switching may cause sharp state transients at switching points. A method to deal with this problem is to make the switching sufficiently slow, so as to allow the transient effects to dissipate after each switching. Based on this idea, the concepts of *dwell time* (DT) and *average dwell time* (ADT) are proposed to tackle the stability and stabilization of switched system, and the reader may refer to [1] and [6] for a survey of state-of-the-art results in this area. The simplest way to specify a slow switching is to introduce a prespecified positive constant $\tau > 0$ and restrict a class of admissible switching signals such that the switching times $t_1, t_2, \ldots$ satisfy the inequality $t_{k+1} - t_k \geq \tau$ for all $k \in \mathbb{N}$. This prespecified positive constant τ is usually called dwell time [5]. Based on concept of dwell time, great effort has been concentrated on the stability, robust stability and supervisory control, etc. see [1,5–8] and the references therein for more details. Besides, the *minimum dwell time* [9] and *persistent dwell-time* [10,11] derived from dwell time have been proposed in the existing literature. The

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minimum dwell time is used to consider the exponential stability of uncertain switched delay systems [9]. The stabilization problem for switched linear systems with additive disturbances is investigated under mode-dependent persistent dwell-time switching [10]. Subsequently, simultaneous control and fault detection for switched delay systems are also investigated under the improved persistent dwell time switching [11]. Actually, the dwell time really does not matter if it occasionally has a smaller dwell time between switchings, provided this does not occur too frequently. This situation is captured by the concept of ADT [12], and the ADT switching can cover the dwell time switching [1]. Therefore, the stability of switched systems with ADT is of both theoretical and practical significance, and some excellent results have been obtained [13,14]. These results have motivated the developments for control (see [15–21]) and stability (see [22–26]) of various classes of switched systems under ADT switching scheme. However, the computation of ADT needs to set two common parameters for Lyapunov-like functions: one is the increase coefficient at switching instants and the other is the decay rate during the running time of subsystems, and ADT switching is independent of the system modes and is still not anticipated [27].

In [25], a notion of *mode-dependent average dwell time* (MDADT) is proposed which can relax the restrictions of ADT. The switching signal with MDADT property only requires that the average time among the intervals associated with each mode is larger than a positive scalar, and the scalars for all subsystems may vary from one subsystem to another. Then, the issues of stabilization [28,29], model predictive control [30] and fault detection [31] are investigated for switched linear systems with MDADT. In summary, the MDADT switching has the advantage of flexibility for a switched system with respect to ADT [27]. Moreover, stability analysis for switched systems composed of unstable subsystems is also studied in [28] by using MDADT switching.

Here, a question arises: can we improve stability conditions for slowly switched systems by further relaxing the requirements of switching signals? Note that a subsystem may switch to other different ones when a switching occurs, and this forms different admissible transition edges. So, every switching from a subsystem to another one will generate a directed edge, and all directed edges derived from switchings among all subsystems can be described as a directed graph. Therefore, how to relax the existing results on switched systems by using *admissible edge-dependent average dwell time* (AED-ADT) switching is worthwhile to be pursued.

Motivated by the above observations, this paper formulates a novel concept of AED-ADT which characterizes a new kind of switching signals for switched systems, and investigates the issues of stability and stabilization for switched linear systems with AED-ADT switching. The results of [27] are further relaxed under the proposed AED-ADT switching. The contributions and advantages of this paper are that a new notion of AED-ADT is firstly proposed, which generalizes the MDADT regime and relaxes the restrictions of MDADT. The proposed AED-ADT switching is more general and contains a larger set of switching signals than MDADT switching. So, the MDADT switching is one special case of AED-ADT switching. Moreover, the stability and stabilization conditions for switched linear systems with AED-ADT switching are also developed in this paper.

The remainder of this paper is organized as follows. In Section 2, a directed switching graph is used to describe the structure of switched system, and some preliminaries are provided for later developments. The main results are stated in Section 3, where the concept of AED-ADT is proposed, and the problems of stability and stabilization of switched linear systems under AED-ADT switching are solved. In Section 4, the effectiveness of the main results is illustrated by a numerical example, and some conclusions are summarized in Section 5.

Notations: Throughout this paper, if not explicitly stated, matrices are assumed to have compatible dimensions. $\mathbb{R}$ $\left(\mathbb{R}^{+}\right)$ is the set of all real (positive real) numbers. $\mathbb{R}^{n}$ and $\mathbb{R}^{n\times m}$ are the n-dimensional real vector space and a set of all $(n \times m)$-dimensional real matrices, respectively. The notation $P > 0\ (\geq 0)$ is used to denote a real positive definite symmetric (semi-positive definite symmetric) matrix. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of matrix P. I represents identity matrix in a block matrix and $\mathbb{N}$ denotes the set of the natural numbers.

2. General model and preliminaries

This section presents some definitions and preliminary results which will be used later. Consider a class of switched linear systems given by

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \tag{1}$$

where $x \in \mathbb{R}^{n}$ is the system state. $u \in \mathbb{R}^{m}$ stands for the control input. The switching signal $\sigma(\cdot) : \mathbb{R}^{+} \to \Phi$ is a piecewise constant function which determines an activated subsystem and takes its value in a finite set $\Phi = \{1, 2, \ldots, M\}$, where M is the number of subsystems. For a switching sequence $t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots (k \in \mathbb{N})$, the ith subsystem is activated during time interval $[t_k, t_{k+1})$ if $\sigma(t_k) = i \in \Phi$, where t_k is the kth switching time instant and $t_0 = 0$ is initial time. Matrices A_i and B_i are assumed to be known and with appropriate dimensions for any $i \in \Phi$.

Remark 1. In fact, many physical systems are with the form of model (1) such as position servomechanism system [32] and drinking water supply network [33]. So, the further development in stabilization theory of switched system (1) is important, and the work of this paper has good application perspective.

In this paper, the transitions from one system to another are represented by walks on a directed switching graph $\mathcal{G}(\Phi, \mathcal{E}(\Phi))$ which consists of the admissible transitions among the subsystems of (1) [34], where the set of vertices of the graph is the set of indices Φ and the set of edges, $\mathcal{E}(\Phi) = \{\mathcal{E}(i,j)\,|\forall i,j \in \Phi, i \neq j\}$. The notation $\mathcal{E}(i,j)$ denotes a