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Stabilization of positive switched delay systems with all modes unstable

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ABSTRACT

This paper studies the stabilization problem of positive switched delay systems (PSDSs) with all modes unstable. Multiple discretized co-positive Lyapunov–Krasovskii functionals are first introduced, by which a delay-dependent sufficient condition for global uniform asymptotic stability of continuous-time PSDSs is provided. It is shown that the state divergence generated by unstable subsystems can be validly compensated via dwell time switching. Moreover, the corresponding results are extended to the case of discrete-time PSDSs. Finally, a numerical example is given to illustrate the effectiveness of the theoretical results.

1. Introduction

As a remarkable class of systems, positive systems [1,2] have been extensively researched in recent years due to their widespread applications in various areas such as communication, economics, sociology, biomedicine and other industries. In particular, positive switched systems [3,4], which consist of a group of positive systems and a switching signal specifying the switching rules, have attracted considerable interests in the field of control. Evidence to date indicates that more and more experts and scholars have began to study the stability problem of positive switched systems, see [5–12]. The methods of common co-positive Lyapunov function [3,4], multiple co-positive Lyapunov functions [7], co-positive polynomial Lyapunov function [11] and joint linear co-positive Lyapunov function [12] are effectively adopted to investigate the stability problems of positive switched systems.

The phenomenon of time delay is common and inevitable in practical systems. Generally, delay systems have complex structures which lead to complicated dynamic characteristics. The effects of time delay on system dynamics could result in the performance degradation or improvement. Up to now, increasing attentions have been paid to the stability problem of positive switched delay systems (PSDSs). In [13], the multiple co-positive type Lyapunov–Krasovskii functionals were first proposed to solve the stability problem of linear positive switched systems with constant delay. In addition, the authors in [14–16] investigated the influence of time-varying delay on the stability of the considered positive switched systems. It should be pointed out that all the above results were about positive switched systems with all modes stable.

Recently, several results concerned with the case that part of the subsystems are not stable, such as [17,18]. The central idea of these results is that the stable subsystems are activated with sufficient runtime to counteract the state divergence generated by unstable subsystems. In other words, the stability of the whole positive switched system is guaranteed by the existence of (at least one) stable subsystems. Obviously, if all subsystems are unstable, the above mentioned idea would be

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ineffective. In order to solve the problem, it is necessary to look for other effective methods. In [19], two ingenious switching laws were provided for discrete-time linear positive switched systems which have neither a stable convex combination nor any stable subsystems. Unfortunately, the method in [19] imposed strict limitations on system matrices, and was also hard to be applied to study the stability problem of PSDSs.

In fact, the switching behavior can stabilize the PSDS with all subsystems unstable under some suitable circumstances. Generally, switching signals are basically divided into two types: state-dependent switching signals [20] and time-dependent switching signals [21,22]. It is worth noting that the state-dependent switching strategies must rely on the current information of the system states. In addition, the corresponding design cost is also an important issue. In terms of time-dependent switching signal, there is no need to consider those issues. For example, the authors in [22] studied the (general not positive) switched continuous-time (delay-free) systems with all subsystems unstable via dwell time switching. The method of discretized Lyapunov function used in [22] can be generalized to positive switched systems. To the best of our knowledge, there are no published literatures focusing on the stabilization of PSDSs with all subsystems unstable.

In this paper, we aim to consider the stabilization problem of PSDSs with all modes unstable. The main contributions of our work lie in:

(1) The stabilization problem for continuous-time positive switched system with time-varying delay and all modes unstable is first investigated.
(2) Multiple discretized co-positive Lyapunov–Krasovskii functionals are constructed as a first attempt, to derive the delay-dependent sufficient conditions for global uniform asymptotic stability of PSDSs with all subsystems unstable.
(3) Compared with the state-dependent switching signal [21], the dwell time switching behavior can stabilize PSDS composed fully of unstable modes without considering the current information of the system states. In addition, it can also greatly reduce the control cost.
(4) The corresponding counterparts for discrete-time PSDS with all subsystems unstable are provided. Furthermore, a delay-independent stability criterion for discrete-time PSDS is given.

The remainder of the paper is organized as follows. Some necessary preliminaries are introduced in Section 2. The stability analysis for continuous-time PSDS and discrete-time PSDS with all subsystems unstable are presented in Section 3. A numerical example is given in Section 4 to verify the obtained theoretical results, and Section 5 concludes this work.

The notations used in this paper are fairly standard. $\mathbb{N} = \{1, 2, 3, \ldots\}$, $\mathbb{N}_0 = \{0, 1, 2, 3, \ldots\}$. For any $p \in \mathbb{N}$, $\underline{p} = \{1, 2, \ldots, p\}$, $\underline{p}_0 = \{0, 1, 2, \ldots, p\}$. The sets of (nonnegative) real numbers and integers are denoted by $\mathbb{R}$ ($\mathbb{R}_+$) and $\mathbb{Z}$, respectively. $\mathbb{R}^n$ ($\mathbb{R}^n_+$) and $\mathbb{R}^{n\times n}$ denote the set of n-dimensional (nonnegative) vectors, and the set of $n \times n$-dimensional real matrices, respectively. A matrix $A = (a_{ij})_{n\times n}$ is said to be Metzler matrix if $a_{ij} \in \mathbb{R}_+$ for any $i \neq j$, $i, j \in \underline{n}$. $\mathbf{M}_n$ stands for the set of Metzler matrices. I_n is the $n \times n$-dimensional identity matrix. $A \succeq 0$ ($\preceq 0$, $\succ 0$, $\prec 0$) denotes that all elements of matrix A are nonnegative (non-positive, positive, negative). A^T stands for the transpose of matrix A. $\|\cdot\|$ represents the Euclidean norm. $\lfloor x\rfloor = \max\{n \in \mathbb{Z} \mid n \leq x, x \in \mathbb{R}\}$.

2. Preliminaries

Consider the following continuous-time PSDS

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t - \omega(t)), \\ x(t) = \phi(t), \quad t \in [-\hat{\omega}, 0], \end{cases} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n_+$ is the state vector, the switching signal $\sigma(t) \in \underline{p}$ is a piecewise constant function and continuous from the right, p is the number of subsystems. The switching time sequence can be described as $0 = t_0 < t_1 < \cdots < t_j < \cdots < +\infty$, the dwell time $\tau_j = t_j - t_{j-1} \in [\tau_{\min}, \tau_{\max}]$, and $\tau_{\min} = \inf_{j\in\mathbb{N}}\tau_j$, $\tau_{\max} = \sup_{j\in\mathbb{N}}\tau_j$, $0 < \tau_{\min} \leq \tau_{\max}$. The matrices A_i and B_i ($i \in \underline{p}$) are the given constant system matrices with appropriate dimensions. $\omega(t)$ is the time-varying delay with $0 \leq \omega(t) \leq \hat{\omega}$ and $\dot{\omega}(t) \leq d < 1$, $\hat{\omega}$ and d are known constants. $\phi(t) : [-\hat{\omega}, 0] \to \mathbb{R}^n_+$ is a continuous and differential initial function.

Definition 2.1 (*[14]*). System (2.1) is said to be positive if for any initial condition $\phi(t) \succeq 0$, $t \in [-\hat{\omega}, 0]$ and any switching signal $\sigma(t)$, the corresponding trajectory $x(t)$ satisfies $x(t) \succeq 0$ for any $t \geq 0$.

Lemma 2.2 (*[14]*). *System (2.1) is positive if and only if $A_i \in \mathbf{M}_n$ and $B_i \succeq 0$ hold for any $i \in \underline{p}$.*

The discrete-time PSDS is described as follows:

$$\begin{cases} x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}x(k - \varpi(k)), \\ x(k) = \varphi(k), \quad k = -\hat{\varpi}, -\hat{\varpi} + 1, \ldots, -1, 0, \end{cases} \tag{2.2}$$

where the state vector $x(k) \in \mathbb{R}^n_+$, $\sigma(k) \in \underline{p}$ is a switching signal, and $0 = k_0 < k_1 < \cdots < k_j < \cdots < +\infty$ is the switching time sequence with the dwell time $\kappa_j = k_j - k_{j-1} \in [\kappa_{\min}, \kappa_{\max}]$, and $\kappa_{\min} = \inf_{j\in\mathbb{N}}\kappa_j$, $\kappa_{\max} = \sup_{j\in\mathbb{N}}\kappa_j$, $0 < \kappa_{\min} \leq \kappa_{\max}$. The time-varying delay $\varpi(k) \in \mathbb{N}_0$ satisfies $\varpi_1 \leq \varpi(k) \leq \varpi_2$, ϖ_1, ϖ_2 are known positive integers, and $\hat{\varpi} = \max\{\varpi_1, \varpi_2\}$. The initial function $\varphi(k) \in \mathbb{R}^n_+$.