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Asynchronous H_∞ control for discrete-time switched systems under state-dependent switching with dwell time constraint

Hong Sang [a], Hong Nie [b,*]

[a] School of Information and Control Engineering, Liaoning Shihua University, Fushun 113001, China
[b] School of Sciences, Liaoning Shihua University, Fushun 113001, China

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ABSTRACT

This paper is concerned with the asynchronous H_∞ control problem for a class of discrete-time switched systems under dwell time constraint. A state-dependent switching strategy that obeys a dwell time constraint is constructed by using an improved dwell-time dependent Lyapunov function (DTDLF). More incisively, this switching strategy not only makes the considered systems stable with a weighted l_2-gain, but also allows the value of DTDLF to increase at the switching instant. Dwell-time dependent asynchronous H_∞ controllers are designed by solving a set of linear matrix inequalities. It has been shown that the obtained results also facilitate the studies of stability, l_2-gain analysis for discrete-time switched systems under zero control input and synchronous switching control, respectively. Finally, three simulation examples are provided to illustrate the effectiveness and potential of the proposed method.

1. Introduction

Switched systems, as an important class of hybrid systems, have attracted considerable attention in recent years due to great theoretical significance and broad practical applications in nature, engineering, and social sciences [1]. A typical switched system is composed of a family of continuous-time or discrete-time subsystems and an associated switching law that orchestrates the switching among them, in which the switching law plays a nontrivial role in the stability and other performance analysis of switched systems. In the past decades, many valuable achievements have been made in the research of switched systems [2–9], where the developed switching strategies are generally classified into two categories, i.e., arbitrary switching [2,3] and constrained switching which includes state-dependent switching [4,5], time-dependent switching [6,7] and random switching [8–10], etc. A common Lyapunov function for all subsystems was proved to be a powerful tool to address the stability of switched systems under arbitrary switching [11]. When switched systems do not possess a common Lyapunov function, multiple Lyapunov functions are always used to design constrained switching laws to ensure the stability of switched systems. And some efficient approaches based on the multiple Lyapunov function theory have also been presented to deal with the case that partial subsystems are unstable [12,13].

The state-dependent switching law was introduced for the stabilization of linear switched systems in continuous-time context [14,15]. As pointed out by [16], this switching law does not guarantee any minimal dwell time, so there generally exists the possibility of fast switching under a state-dependent switching mechanism. Thus, a well-defined switching signal is required, which means a finite number of switches in any finite time interval [17]. For example, the allowable switching

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[*] Corresponding author.

E-mail addresses: sanghong@lnpu.edu.cn (H. Sang), niehong@lnpu.edu.cn (H. Nie).

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frequency in digital networks is restricted by a data rate limit. In this case, a dwell time constraint is necessary and non-negligible in the design of stabilizing switching signal for a switched system. Some recent results proposed a combined switching strategy which incorporates the state-dependent switching with the time-dependent switching [16,18,19]. This strategy possesses the advantages of both slow switching and state-dependent switching. Among these results, the stability and L_2-gain analysis were reported for linear switched systems with dwell time and polytopic type parameter uncertainty in [16], while [18] considered the passive analysis and output feedback passification for a class of continuous-time switched systems, and the stability and weighted L_2-gain analysis were addressed for the switched linear systems under state and dynamic output feedback control in [19]. However, these existing results require that the Lyapunov function is nonincreasing at the switching instant. On the other hand, by allowing the Lyapunov function to increase during the dwell time portion, [20] addressed the issues of stability analysis and control for the switched linear system with unstable subsystems under dwell time constraint, and the potential increment of the Lyapunov function will be compensated by the "drop" at each switch instant. Moreover, the discrete-time counterpart was also discussed in [21].

It should be noted that the above-mentioned results using a combined switching strategy are developed under the premise that the controllers are switched synchronously with the system modes, which is an idealized situation. The practical case, however, is one where the asynchronous switching between the system modes and the controllers is common since it usually takes some time to identify the activated subsystem and then to activate the corresponding controller [22]. For a review of the stability and other performance analysis of switched systems under asynchronous switching, see [23–26]. So it is necessary to investigate the asynchronous switching control issue for a class of switched systems via combined switching strategy. Recently, H_∞ control and filtering problem have been reported for discrete-time switched systems under dwell-time constraint and asynchronous switching [27,28]. But they require that the switching instant can be detected online and the systems must switch to the subsystem yielding a smaller value for the Lyapunov function. In fact, this is usually hard to check in its full generality [11]. As far as we know, under the combined switching strategy, asynchronous H_∞ control problem for discrete-time switched systems has not been reported with considering that the Lyapunov function can be increasing. This motivates our research.

This paper mainly addresses the asynchronous H_∞ control problem for a class of discrete-time switched systems by application of state-dependent switching strategy incorporated with time-dependent switching strategy. The main contributions can be summarized as follows: (i) An improved dwell-time dependent Lyapunov function (DTDLF) is constructed. Under the DTDLF, a novel combined switching strategy is presented such that the resulting switching signal possesses a dwell time constraint between consecutive switching. (ii) The proposed DTDLF, which depends on both the previous active subsystem and the current active subsystem, is allowed to increase during the unmatched interval and at the switching instant in contrast with other existing results. (iii) The conditions of the existence of time-dependent switching controllers are derived to guarantee the stability and l_2-gain performance for the switched systems in the cases of synchronous and asynchronous switching. The obtained results contain the asynchronous H_∞ control issue for discrete-time linear switched systems with dwell time constraint in [27] as a special case. Finally, a numerical example, PWM (Pulse-Width-Modulation)-driven boost converter and three-tank examples are provided to show the effectiveness and potential of the developed results.

The remainder of this paper is organized as follows. Section 2 gives problem formulation and preliminaries. In Section 3, we present the definition of improved dwell-time dependent Lyapunov function (DTDLF). Section 4 is devoted to deriving the main results on stability and l_2-gain analysis for the considered discrete-time switched systems. In Section 5, three examples are provided to illustrate the validity of the obtained results. Section 6 draws the conclusion.

Notation. Throughout this paper, the notations are standard. The superscript T stands for matrix transposition. In symmetric matrix or long matrix expressions, we use a symbol $*$ to represent a term that is induced by symmetry. $\mathbb{R}^n$ stands for the n dimensional Euclidean space while $\mathbb{R}^{n\times l}$ stands for the set of all $n \times l$ real matrices. $\mathbb{N}$ denotes the set of natural numbers. $\mathbb{Z}$ denotes the set of integers and for any given set $\mathcal{M} \subset \mathbb{R}$, $\mathbb{Z}_{\mathcal{M}} = \mathbb{Z} \cap \mathcal{M}$. $\|\cdot\|$ refers to the standard Euclidean vector norm and $l_2[0,\ \infty)$ is the space of square summable sequence and for $\omega(k) \in l_2[0,\ \infty)$, its norm is given by $\|\omega\|_2 = \sqrt{\sum_{k=0}^{\infty}|\omega(k)|^2}$.

2. Problem formulation and preliminaries

Consider the following discrete-time switched systems

$$\begin{cases} x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k) + D_{\sigma(k)}\omega(k), \\ z(k) = C_{\sigma(k)}x(k) + E_{\sigma(k)}u(k) + F_{\sigma(k)}\omega(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^p$ is the control input, $\omega(k) \in \mathbb{R}^q$ is the exogenous disturbance input which belongs to $l_2[0,\infty)$, $z(k) \in \mathbb{R}^r$ is the controlled output. $\sigma(k) : \{0,\ 1,\ 2\cdots\} \to M = \{1, 2, \ldots, m\}$ is a piecewise constant function of time k called switching signal. $\sigma(k) = i$ means that the ith subsystem is activated. $m > 1$ is the number of subsystems. A_i, B_i, D_i, C_i, E_i, F_i are known constant matrices with appropriate dimensions.

The switching signal $\sigma(k)$ can be characterized by the switching sequence

$$\Sigma = \{x_0 : (i_0, k_0), (i_1, k_1), \ldots, (i_l, k_l), (i_{l+1}, k_{l+1}), \ldots | i_l \in M, l \in \mathbb{N}\} \tag{2}$$