Nonlinear Analysis: Hybrid Systems 29 (2018) 234–246

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Adaptive finite-time control of a class of Markovian jump nonlinear systems with parametric and dynamic uncertainties

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A R T I C L E   I N F O

Article history:
Received 24 April 2017
Accepted 16 February 2018

Keywords:
Nonlinear system
Markovian jump system
Finite-time globally asymptotical stability
Adaptive control

A B S T R A C T

In this paper, we investigate the finite-time stabilization of Markovian jump nonlinear systems with parametric and dynamic uncertainties. Firstly, a proper criteria on finite-time globally asymptotical stability in probability (FGSP) and some useful lemmas are introduced. Then, overcoming the common influence of parameter uncertainties and coupled item which determined by Markovian switching, a state-feedback finite-time controller is explicitly constructed by adding a power integrator technique and induction method. It is proven that, the system state of the closed-loop systems is FGSP. Simulation example illustrates the effectiveness of our method.

1. Introduction

In this paper, we consider the finite-time stabilization control problem of a class of Markovian jump nonlinear systems with parameters in the form of

$$\begin{aligned} \dot{x}_1 &= [x_2]^{r_1} + f_1(x_1, r(t), \theta), \\ \dot{x}_2 &= [x_3]^{r_2} + f_2(x_1, x_2, r(t), \theta), \\ &\ \vdots \qquad\qquad\qquad\qquad\qquad\qquad t \geq 0, \\ \dot{x}_n &= [u]^{r_n} + f_n(x_1, x_2, \ldots, x_n, r(t), \theta), \end{aligned} \tag{1}$$

where $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$ and $u \in \mathbb{R}$ are the system state and control input, respectively. $r(t)$ is a continuous-time discrete-state Markov process taking values in a finite set $\mathcal{S} = \{1, 2, \ldots, N\}$ with transition probability matrix $P = \{p_{ij}\}$ given by $p_{ij} = Pr(r(t+\Delta) = j \mid r(t) = i) = \begin{cases} \pi_{ij}\Delta + o(\Delta), & i \neq j; \\ 1 + \pi_{ii}\Delta + o(\Delta), & i = j, \end{cases}$ $\Delta > 0$. Here $\pi_{ij} \geq 0$ is the transition rate from i to j $(i \neq xj)$, $\pi_{ii} = -\sum_{j=1, j\neq i}^{N} \pi_{ij}$, and $o(\Delta)$ is an infinitesimal of higher order than Δ. For $i = 1, \ldots, n$ and any $r \in \mathcal{S}$ and any parameter θ, f_i are continuous functions with $f_i(0, \ldots, 0, r, \theta) = 0$. The powers r_i which appeared in the nonlinearity $[x_{i+1}]^{r_i}$ satisfy $r_i > 0, i = 1, \ldots, n$ and $[\cdot]^r = sign(\cdot)|\cdot|^r, r > 0$.

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https://doi.org/10.1016/j.nahs.2018.02.003
1751-570X/

Many physical systems are subject to frequent unpredictable structural changes, such as random failures, sudden environment disturbances and abrupt variation of the operating point on a nonlinear plant. Markovian jump systems are often used to describe such systems. The stability and control of this class system have recently received a lot of attention. For example, Ji, & Chizeck [1] and Mariton [2] studied the stability of linear jump equations, separately. Mao [3] investigated the exponential stability of general nonlinear stochastic differential equations with Markovian switching. Zhao [4] investigated the practical stability of this class system with time-delays. In the area of control for this class system, the linear quadratic regulator problem is studied by Sworder [5], Wonham [6] and Mariton and Bertrand [7] through state feedback and output feedback, respectively. For the optimal control problem of general stochastic differential equations with Markovian switching, Ghosh et al. [8] have developed a dynamic programming approach. The robust stability and stabilization of jump systems have also been studied by many scholars, such as Yuan and Lygeros [9] and Wang et al. [10]. Especially, a backstepping controller is designed for a class of Markovian jump stochastic nonlinear systems by Wu et al. [11]; a finite-time controller is also designed for a class of Markovian jump stochastic nonlinear systems by Zhao et al. [12]. In this paper, for a more general Markovian jump nonlinear systems, a finite-time controller will be designed. The method on dealing with the coupled items induced by the Markovian jump parameters are new, which make the controller design more easier for Markovian jump system with unknown parameters.

For the study of non-smooth finite-time control, it has drawn increasing attention in the last years. It can provide, in some sense, fast response and high tracking precision as well as disturbance rejection properties because of their non-smoothness [13]. One of the main benefits of the nonsmooth finite-time control strategy is that it can force a control system to reach a desirable target in finite time. This approach was first studied in the literature of optimal control. Despite its potential application to practical problems, the study of finite-time stabilization is quite underdeveloped, partially because of the lack of effective and constructive tools in non-smooth analysis. In recent years, finite-time stabilizing controllers were constructed for some classes of deterministic nonlinear systems [13–18]. For stochastic systems, finite time stabilization have not been studied fully. In [19–22], the definition of finite-time stability is provided and some criteria have been given by Chen and Yin, etc. Only in [23,24] etc., finite-time stabilization of a class of stochastic nonlinear systems was discussed. For Markovian jump systems, the non-smooth finite-time control problem has rarely been considered.

For system (1) without Markovian jump parameters, when the powers $r_i = 1, i = 1, \ldots, n$, the finite time stabilization problem has received considerable attention in many papers (see, e.g., [25–28]). When $r_i = r \in (0, 1)$ is a ratio of odd integers, [29] explicitly construct a dynamic output feedback controller by extending the adding-a-power-integrator technique for a class of continuous but nonsmooth nonlinear systems. When $r_i \geq 1$, the stabilization of system (1) will become much difficult and has been widely known as a challenging problem. As described in [30], it is usually not solvable by any smooth feedback, even locally, because of the inherent nonlinearity exhibited in system (1). In this case, nonsmooth feedbacks are naturally introduced for dealing with the control design problem. For example, by using the adding an integrator technique, a sufficient condition for nonsmooth stabilization of triangular systems is derived in [31]. In [17,32,33], when r_i are odd positive integers, nonsmooth finite-time controller is designed for p normal form nonlinear systems with parametric uncertainty. For a more general case, the powers $r_i > 0$ which contains the two cases: $0 < r_i < 1$ and $r_i \geq 1$, Ding et al. [34] design a nonsmooth finite-time controller.

In this paper, the finite-time stabilization control of Markovian jump nonlinear systems with parametric and dynamic uncertainties will be studied. The system is in a p-normal form, and the powers $r_i > 0$ involve two cases: $0 < r_i < 1$ and $r_i \geq 1$. Firstly, a proper criterion on finite-time globally asymptotical stability in probability (FGSP) is introduced. Then, by adding a power integrator technique and induction method, overcoming the common influence of parametric uncertainties and Markovian switching, a state-feedback finite-time controller will be explicitly constructed. By the method of adding zero items, the coupled item, induced by Markovian jump parameters, is eliminated directly. This makes the finite-time stabilization of Markovian jump systems with unknown parameters easier. It is proven that, the equilibrium at the origin of the closed-loop systems is bounded and the system state is FGSP.

The remainder of this paper is organized as following: Section 2 provides some notations and introduces the definitions of FGSP and some useful lemmas. Section 3 explicitly constructs a state feedback finite time controller which makes the system state reaches zero in finite time. In Section 4, a simulation example is provided to illustrate the results. Section 5 includes some concluding remarks.

2. Notations and preliminary results

Throughout this paper, $\mathbb{R}_+$ denotes the set of all nonnegative real numbers; $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, n-dimensional real space and $n \times m$ dimensional real matrix space. For vector $x \in \mathbb{R}^n$, $|x|$ denotes the Euclidean norm $|x| = (\sum_{i=1}^{n} x_i^2)^{1/2}$. All the vectors are column vectors unless otherwise specified. The transpose of vectors and matrices are denoted by superscript T. $\mathcal{C}^i$ denotes all the ith continuous differential functions; $\mathcal{C}^{i,k}$ denotes all the functions with ith continuously differentiable first component and kth continuously differentiable second component. $E\{x\}$ denotes the expectation of stochastic variable x.

A function h: $\mathbb{R}_+ \to \mathbb{R}_+$ is said to belong to the class generalized $\mathcal{K}$ ($\mathcal{GK}$) if it is continuous with $h(0) = 0$, and satisfies

$$\begin{cases} h(r_1) > h(r_2), & \text{if } h(r_1) \neq 0; \\ h(r_1) = h(r_2) = 0, & \text{if } h(r_1) = 0, \end{cases} \forall r_1 > r_2.$$