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Controller synthesis for switched T–S fuzzy positive systems described by the Fornasini–Marchesini second model

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ABSTRACT

In this paper, controller synthesis problem is investigated for a class of Fornasini–Marchesini second type switched systems with time-varying delays under the Takagi–Sugeno fuzzy rules. Both the cases of state feedback controller and static output feedback controller are designed. By utilizing the common Lyapunov function approach and the multiple Lyapunov functions method, sufficient criteria are derived in the form of linear matrix inequalities to assure the resulting closed-loop 2-D systems to be positive and asymptotically stable, which can be easily computed using the existing numerical algorithms. Explicit design schemes for the state feedback/static output feedback controllers gain matrices are also presented. Finally, advantage and effectiveness of the proposed criteria are shown by a numerical example.

1. Introduction

As is well known that plenty of practical systems have positive behavior, i.e., the state variables and outputs of these systems take only nonnegative values whenever the initial conditions and inputs are nonnegative [1]. Such kind of systems are referred to as positive systems [2–4], whose domain is obviously a cone — the first quadrant of the n-dimensional Euclidean space [5]. Considering the characteristic of positive systems, one might choose the co-positive Lyapunov functions instead of the traditional quadratic ones when discussing the stability issues, where less conservative conditions might be obtained via the former ones [6,7]. On another front, two-dimensional (2-D) systems are introduced to better understand and describe the behaviors of those processes and devices whose evolutions depend on two independent variables [8], such as those in the river pollution modeling, process of gas filtration, grid-based wireless sensor networks, and data processing [9]. In the literature, the most popular 2-D linear systems proposed by Roesser [10], Fornasini–Marchesini [11] and Kurek [12] have been extensively studied, see [13–16] for example. In recent years, these models have been expanded to uncertain systems [17], Markovian jump systems [18], Takagi–Sugeno (T–S) fuzzy systems [19] and so on to incorporate appropriate applications.

To deal with the nonlinear systems which are usually hard to be analyzed explicitly, some powerful approaches have been developed, for instance, backstepping control, adaptive control, etc. [20,21]. By combining the adaptive control with the backstepping technique, the issue of state-feedback decentralized stabilization control and the adaptive tracking control problem have been discussed, respectively, in [22] and [23] for a class of systems with nonlinear functions which satisfy the local Lipschitzian condition. In order to investigate more general complex nonlinear systems which do not meet the local Lipschitzian restriction, the Takagi–Sugeno (T–S) fuzzy model approach introduced by Takagi and Sugeno in 1985 [24] has

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been created, which combines the rigorous mathematical theories on linear systems and the flexibility of fuzzy logic theories into a unified framework [25]. In fact, T–S fuzzy model is a weighted sum of linear systems which can be analyzed by using some conventional linear system theories, and great attention has been paid to the stability analysis and controller synthesis of these systems, see [26,27] and the references cited therein for more details. It should be noted that T–S fuzzy model has a good approximation capability for the highly nonlinear functions, which could approximate any smooth nonlinear function to any degree of accuracy in the convex compact regions [28,29].

Generally, time-delays are unavoidable in practical systems due to the finite speed of signal transmission, and the switching phenomena are also taken into account because many practical systems are always subjected to abrupt changes in their structures and parameters [30,31]. Both time delays and switching are often the source of instability, in other words, they both have great influence on the stability of systems. Therefore, the stability analysis issue for switched systems with time-delays is also a hot research topic (see [32–34] and the references cited therein for example). If there exists a common Lyapunov function (CLF) for all subsystems, then the switched system retains its stable characteristic under arbitrary switchings. However, for most switched systems in practice, they do not possess a CLF but the stability can still be maintained by using the multiple Lyapunov functions (MLFs) approach [35,36]. As is well recognized, MLFs approach can provide a larger region of feasibility than the CLF method, i.e., conservatism of the stability criteria can still be further reduced via the MLFs approach.

It is worth mentioning that majority of the results in the literature are focused on one-dimensional (1-D) systems. For example, the stability problem and l_1 controller synthesis are considered in [37,38] for the 1-D fuzzy positive systems. In [39], via the co-positive CLF and co-positive MLFs approaches, respectively, robust stability is analyzed for the 1-D switched positive systems with time-varying delay. As for the 1-D switched positive nonlinear systems, local stability issues have been investigated in [40], which have been extended to the global case in [41] by using the approximation property of T–S fuzzy rules. Unfortunately, up to now, there are few related works for 2-D switched T–S fuzzy positive systems, which are mainly due to the difficulties in analyzing/computing the 2-D models.

Motivated by the above discussions, in the current paper, controller synthesis problem is studied for a class of Fornasini–Marchesini second (FM-II) type switched systems under the T–S fuzzy rules. *Compared with the existing results, the main advantages of the present paper can be summarized as three folds. (1) By resorting to the T–S fuzzy model approximation method, the stabilization problem for complex 2-D nonlinear systems can be solved. (2) Both MLFs and CLF methods are used to analyze the stability of the closed-loop systems resulting, respectively, from state feedback and static output feedback. In addition, conservatism of the corresponding results are further compared by the numerical example presented in* Section 5. *(3) By making full use of the properties of Moore–Penrose generalized inverse, the state feedback fuzzy controller and the static output feedback fuzzy controller are explicitly designed to guarantee the resulting closed-loop systems to be positive and asymptotically stable.* The remainder of this paper is organized as follows. Section 2 presents the problem formulation and some preliminaries. Section 3 considers the case of state feedback control. To be more specific, explicit state feedback controllers are designed, respectively, via the MLFs method and the CLF approach, under which the closed-loop 2-D fuzzy system with time-varying delays is positive and asymptotically stable. The case for the static output feedback control scheme is discussed in Section 4. One numerical example provided in Section 5 demonstrates the effectiveness and applicability of the obtained results. The paper is concluded in Section 6.

Notation. The notation used in this paper is fairly standard, $\mathbb{R}^{n\times m}$ stands for the set of all $n \times m$ matrices with entries from the field of real numbers $\mathbb{R}$, and $\mathbb{R}_+^{n\times m}$ is the set of all $n \times m$ matrices with nonnegative entries. $\mathbb{R}^n = \mathbb{R}^{n\times 1}$ denotes the set of n-dimensional column vectors, if further all entries are nonnegative (respectively, positive), it is denoted as $\mathbb{R}_+^n$ (respectively, $\mathbb{R}_{++}^n$) in which the vectors are called nonnegative (respectively, strictly positive) vectors. $X \succeq 0$ (respectively, $\prec 0$) means that all entries of matrix X are nonnegative (respectively, negative), and X^T denotes the transpose of X. I_n represents the identity matrix with $n \times n$ dimension. $\mathrm{diag}\{\cdots\}$ denotes a diagonal matrix and the set of all nonnegative integers will be represented by $\mathbb{Z}_+$. $\mathbf{1}_n$ is an n-dimensional column vector whose elements are all 1, and $[X]_{k,l}$ stands for the element located at the kth row and lth column of matrix X. The 1-norm of a nonnegative vector is defined as $\|x\|_1 \triangleq \sum_{s=1}^n x_s$ for $x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}_+^n$.

2. Problem formulation and preliminaries

Consider the following FM-II type switched fuzzy model with time-varying delays:
Model rule $\mathbf{R}_\varsigma^{\sigma(i,j+1)}$: IF $\bar{\theta}_1(i,j)$ is $M_{\varsigma 1}^{\sigma(i,j+1)}$, ..., and $\bar{\theta}_\iota(i,j)$ is $M_{\varsigma\iota}^{\sigma(i,j+1)}$, THEN

$$\begin{cases} x(i+1,j+1) = A_{1\varsigma}^{\sigma(i,j+1)}x(i,j+1) + A_{2\varsigma}^{\sigma(i+1,j)}x(i+1,j) + A_{1d\varsigma}^{\sigma(i,j+1)}x(i-d_1(i),j+1) \\ \qquad\qquad + A_{2d\varsigma}^{\sigma(i+1,j)}x(i+1,j-d_2(j)) + B_{1\varsigma}^{\sigma(i,j+1)}u(i,j+1) + B_{2\varsigma}^{\sigma(i+1,j)}u(i+1,j), \\ y(i,j+1) = C_\varsigma^{\sigma(i,j+1)}x(i,j+1), \\ y(i+1,j) = C_\varsigma^{\sigma(i+1,j)}x(i+1,j), \end{cases} \tag{1}$$

where $\varsigma \in \underline{\mathbf{r}} \triangleq \{1, 2, \ldots, r\}$ and r is the number of IF–THEN rules; $\bar{\theta}_\hbar(i,j) \triangleq (\theta_\hbar(i,j+1), \theta_\hbar(i+1,j))$ is the spatial premise variable, $M_{\varsigma\hbar}^{\sigma(i,j+1)}$ is the spatial fuzzy set of rule ς corresponding to $\bar{\theta}_\hbar(i,j)$ with $\hbar = 1, 2, \ldots, \iota$; $x(i,j) \in \mathbb{R}^n$ is the state