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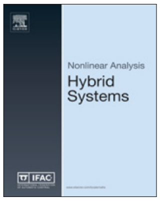

Switching time-optimal control of spacecraft equipped with reaction wheels and gas jet thrusters

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ABSTRACT

This paper studies the time-optimal control problem of a rigid spacecraft equipped with both reaction wheels and gas jet thrusters, in which the reaction wheels are the main actuators and the gas jet thrusters act only after saturation or to prevent future saturation of the reaction wheels. It is assumed that the control torques are generated about the principal axes of the spacecraft. The presence of both reaction wheels and thrusters gives rise to two operating modes for each axis. Since this system can change dynamics, it can be regarded as a switched dynamical system. The time-optimal control problem for this system is solved using the embedding approach. With this technique the switched system is embedded into a larger set of systems and the optimal control problem is formulated in the latter. The main advantages of this technique are that assumptions about the number of switches are not necessary, integer or binary variables do not have to be introduced to model switching decisions between modes, and the optimal values of the switching times between modes are obtained without introducing them as unknowns of the optimal control problem. As a consequence, the resulting problem is a classical optimal control problem. Feasibility of the obtained solution is validated through propagation of the initial state. Optimality of the obtained solutions is verified by checking the compliance with Pontryagin's Maximum Principle.

1. Introduction

In this paper, the time-optimal control problem of a rigid spacecraft equipped with both reaction wheels and gas jet thrusters is studied. Bound constraints on both torque of the actuators and angular momentum of the reaction wheels are taken into account. To cope with the saturation of the reaction wheels, the spacecraft is equipped with thrusters which are able to generate torques about the axes of the reaction wheels. Thus, there are two operating modes for each axis, and, as a consequence, 2^3 operating modes for the spacecraft.

The problem can be stated as follows: given an initial state and a final state, find the sequences of modes, the corresponding trajectory and control inputs that satisfy the dynamic equation of the spacecraft, and steer the system between the initial and the final states minimizing the duration of the maneuver. Since the optimal sequence of modes has to be determined, this problem is actually a time-optimal control problem of a switched dynamical system.

In spite of the apparent similarities between reaction wheels and gas jet thrusters actuations, the corresponding control properties of the system are very different. In the first case, the spacecraft is controlled by momentum exchange devices subject to momentum conservation law, whereas in the second case it is controlled by means of external control torques.

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The most common actuators for spacecraft are gas jet thrusters. The main disadvantage of using gas jet thrusters is fuel consumption. In contrast, although the torque that reaction wheels can offer is typically one order of magnitude lower than that of thrusters, reaction wheels are capable of finer control, and most importantly, they only consume electrical energy. The main disadvantage of reaction wheels is their saturation, that is, since they generate torque on the spacecraft by accelerating, they cannot operate when they reach their maximum angular speed. In this paper, a novel control logic for mixed actuation is investigated, more specifically, the possibility of using reaction wheels as the main actuators, which are supposed to operate with the support of the thrusters that act only after saturation or to prevent future saturation of the reaction wheels. Since the time-optimal control is of bang–bang type and reaction wheels are subject to saturation, the torque on the reaction wheels, their angular momenta, and the torque of the gas jet thrusters have been constrained in this work in order to get practical solutions.

In [1], the necessary and sufficient conditions for the controllability of a rigid body in the case of one, two and three independent control torques are provided. If the spacecraft is controlled by three independent torques it is completely controllable, although in the case of momentum wheel actuators a certain minimum control effort is required.

In general, works on minimum-time spacecraft reorientation [2–7], consider bounds on the control inputs, but do not include the dynamics of spacecraft, reaction wheels or control moment gyros. In these works, the Pontryagin's Maximum Principle is used to derive the optimality conditions to solve the optimal control problem. The Pontryagin's Maximum Principle gives the first-order necessary conditions for the solution of optimal control problems and includes the classical necessary condition given by the Euler–Lagrange equation, a second-order differential equation originally obtained by means of the theory of calculus of variations. The functions that satisfy the first-order necessary conditions are called extremals. However, the first-order necessary conditions of the Pontryagin's Maximum Principle depend on the constraints of the problem and change when some constraints become active, i.e., when the state or the control reach the boundary of their admissible sets. This corresponds to discontinuities of the first derivative of the extremals. Similarly, the Euler–Lagrange equation holds only at those points at which the extremals are smooth and therefore at points where their first derivatives are not continuous, which are called corners, the so-called Weierstrass–Erdmann corner conditions must be satisfied. The main drawback of this setting is that it is not possible to know in advance the number of corners and their location in time, and therefore it is not easy to derive a general numerical method from these conditions. A numerical method to overcome these difficulties has been presented in [8]. It is based on the Euler–Lagrange necessary condition in integral form and has been applied to several optimal control problems with holonomic, nonholonomic and bound constraints in [9] and [10].

In [11], a version of the Maximum Principle for hybrid optimal control problems under weak regularity conditions has been presented. In particular, autonomous systems have been considered in which the dynamical behavior and the cost are invariant under time translations. The Maximum Principle has been stated for both, problems where the dynamics, the Lagrangian, the cost functions for the switchings, and the endpoint constraints are differentiable along the reference arc, and problems involving nonsmooth maps. In [12] the Hybrid Maximum Principle has been stated and necessary conditions for hybrid optimization have been studied. In particular, optimization problems on fixed compact intervals of time have been considered, in which the set of equivalence classes is finite in such a way that it is possible to drive strong conclusions from the Maximum Principle. In [13], necessary conditions of optimality, in the form of a Maximum Principle, have been presented for a broad class of hybrid optimal control problems in which the dynamics takes the form of differential equations with control terms, and restrictions on the switches between operating modes are described by collections of functional equality and inequality constraints. A wide range of possible autonomous and controlled switching strategies have been provided by different choices of the constraint functionals.

Whether the eigenaxis maneuver is optimal or not depends on the definition of the set of admissible control torques. Consider the time-optimal reorientation problem of a rigid inertially symmetric spacecraft. Assume that the control axes are aligned with the principal axes and that the control torques for each axis are bounded. Early works on time-optimal spacecraft reorientation are described in [14]. It has been shown in [2] that eigenaxis rotations, which provide the minimum angular path between two orientations, are not time optimal even in inertially symmetric spacecraft. On the contrary, it has been shown in [3] that for rigid inertially symmetric spacecraft in which the total magnitude of the control torque is constrained, the eigenaxis maneuver is indeed the time-optimal solution.

In [15], the minimum-time reorientation problem of a rigid axisymmetric spacecraft is solved numerically using a direct collocation method [16]. In [17], the pseudospectral technique was employed to solve the time-optimal reorientation problem of nonsymmetric rigid bodies. This technique was applied to more realistic cases of time-optimal reorientation of spacecraft in [18,19,6,20].

It is worth noting that the first orbital time-optimal maneuver performed on a real spacecraft was implemented on the NASA Transition Region and Coronal Explorer (TRACE) telescope in 2010, after twenty years of research on this topic. The design and flight implementation of time-optimal attitude maneuvers of this telescope are described in [21], which considers bounds on control and state variables, and the dynamics of the reaction wheels.

In [22], a time-optimal controller for reorientation maneuver of a spacecraft with saturation constraints on both torque and angular momentum of the reaction wheels is presented. The proposed control scheme consists in generating an open-loop minimum-time reorientation trajectory using the Legendre pseudospectral method which is tracked using a closed-loop control strategy.

In [23], the minimum-time reorientation of a spacecraft with both reaction wheels and gas jet thrusters is considered. First, the problem is studied assuming that only reaction wheels are present. Then, gas jet thrusters are considered together