Nonlinear Analysis: Hybrid Systems 29 (2018) 322–332

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

A linear programming approach for stabilization of positive Markovian jump systems with a saturated single input

In Seok Park, Nam Kyu Kwon, PooGyeon Park *

Department of Electrical Engineering, Pohang University of Science and Technology, Pohang, Gyungbuk 790-784, Republic of Korea

ARTICLE INFO

Article history:
Received 25 July 2017
Accepted 2 March 2018

Keywords:
Linear programming
Positive Markovian jump systems
Input saturation
State-feedback control

ABSTRACT

This paper proposes a linear programming approach for stabilization of positive Markovian jump systems (PMJSs) with a saturated single input. The proposed approach first derives a sufficient condition for stabilization of PMJSs with input saturation based on the linear co-positive Lyapunov function. By introducing an intermediate scalar whose absolute value is less than the absolute value of product of nonnegative vector of the linear co-positive Lyapunov function and input matrix and constructing a special form of the controller gains, this approach obtains a modified condition applicable for the linear programming. Finally, four numerical examples show that the proposed approach gives the larger domain of attraction than the existing approach based on the quadratic Lyapunov function.

1. Introduction

Physical systems often have variables that include nonnegative property: networks of reservoirs, industrial processes involving chemical reactors, the population of human and heat exchangers [1]. Such systems are indicated as positive systems, whose state variables take only nonnegative values for any nonnegative initial condition. More recently, many applications of positive switched systems can be found in practice: virus mutation treatment [2], turbofan engines [3] formation flying [4] and other areas. Consequently, the positive switched systems have become subjects of research interest [5–7]. For positive systems, it has been shown that the linear co-positive Lyapunov function is more valid for discussing the control synthesis than traditional quadratic Lyapunov functions. Accordingly, the linear programming technique is more efficient than the linear matrix inequality (LMI) technique because the number of decision variables in the linear programming conditions is usually far fewer than that in the LMI conditions [8–14].

In the meantime, Markovian jump systems (MJSs) can be modeled by a set of linear systems with mode transition subject to a Markov chain (or Markov stochastic process). Over the past ten years, MJSs have gained a substantial amount of attention due to the fact that they are commonly regarded as suitable mathematical models to describe dynamic systems subject to random abrupt variations in their structure or parameters. For this reason, MJSs have been rapidly developed in many fields: networked control systems [15], economic systems [16], anti-windup design [17] and so on. Moreover, positive Markovian jump systems (PMJSs) are a special class of MJSs which provide a unified framework for mathematical modeling of many dynamic systems such as virus mutation treatment [2], turbofan engines [18,3] and network employing TCP in communication systems [19,20]. The stability analysis of PMJSs also has been studied [8,5,21]. These researches illustrate the necessity of the theoretical findings PMJSs.

On the other hand, input saturation often occurs in practical engineering, which gives a clipped control input that is hard-limited by the peak output of an actuator. Since input saturation may deteriorate the performance of systems, it is

* Corresponding author.
E-mail addresses: ispark90@postech.ac.kr (I.S. Park), kwunnam@postech.ac.kr (N.K. Kwon), ppg@postech.ac.kr (P. Park).

https://doi.org/10.1016/j.nahs.2018.03.001
1751-570X/

necessary to take input saturation into account in stabilization of systems, so that the control synthesis for systems with input saturation have been investigated by many researchers [22–24]. Especially, the authors of [24] considered the LMI approach for stabilization of PMJSs subject to actuator saturation by using the traditional quadratic Lyapunov function. As previously mentioned, the LMI-based strategy may be less effective than the linear programming based strategy. Therefore, it is theoretically meaningful to consider the linear programming approach for stabilization of PMJSs with input saturation by using the linear co-positive Lyapunov function. However, stabilization of PMJSs with input saturation by using the linear co-positive Lyapunov function yields the mutually coupled decision variables, which results in that the condition is not applicable for the linear programming. This difficulty motivates us to carry out this study.

This paper proposes a linear programming approach for stabilization of positive Markovian jump systems with a saturated single input. The proposed approach first derives the sufficient conditions for stabilization of PMJSs with input saturation based on the linear co-positive Lyapunov function. As mentioned earlier, the directly obtained stabilization conditions are not applicable for the linear programming. By introducing an intermediate scalar whose absolute value is less than the absolute value of product of nonnegative vector of the linear co-positive Lyapunov function and input matrix and constructing a special form of the controller gains, this approach obtains a modified condition applicable for the linear programming. Finally, four numerical examples show that the proposed approach gives the larger domain of attraction than the existing approach based on the quadratic Lyapunov function [24].

The notations used in this paper are fairly standard. For $x \in \mathbb{R}^n$, x^T means the transpose of x. I_n means the $n \times n$ identity matrix. $\mathbb{N}_r^+ = \{1, 2, \ldots, r\}$, where r is positive integer. Given a probability space $(\Omega, \Upsilon, \Theta)$, Ω represents the sample space, Υ is the algebra of events, and Θ is the probability measure defined on Υ. $A \succ 0$, which indicates that all the elements of A are positive, and $A \succ B$, which means that $A - B \succ 0$. A matrix A is called a Metzler matrix if its off-diagonal entries are nonnegative. For a vector $v \in \mathbb{R}^n$, define $\epsilon(v) = \left\{x \in \mathbb{R}^n \;\; x^T v < 1\right\}$. For a matrix $\mathrm{A} \in \mathbb{R}^{m\times n}$, $(A)_{pq}$ is used to indicate the entry in the p_{th} row and q_{th} column of the matrix A, where $p \le m$ and $q \le n$. For a matrix $H \in \mathbb{R}^{m\times n}$, $L(H) = \left\{x \in \mathbb{R}^n \;\; |h_i x| \le 1,\; i \in N_m^+\right\}$, h_i denotes the ith row of H.

2. Problem statement

Given a probability space $(\Omega, \Upsilon, \Theta)$, consider a continuous-time positive Markovian jump systems with input saturation given by

$$\dot{x}(t) = A(r_t)x(t) + B(r_t)sat(u(t)), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input. $\{r(t), t \ge 0\}$ is a continuous-time Markov process on the probability space that takes the values in a finite set $\mathbb{N}_N^+ = \{1, 2, \ldots, N\}$ and has the mode transition probabilities

$$\Pr(r_{t+\delta t} = j | r_t = i) = \begin{cases} \pi_{ij}\delta t + o(\delta t) & if \;\; j \neq i, \\ 1 + \pi_{ii}\delta t + o(\delta t) & otherwise, \end{cases} \tag{2}$$

where $\delta t > 0$ and $\lim_{\delta t\to 0}(o(\delta t)/\delta t) = 0$. π_{ij} is the transition rate from mode i at time t to mode j at time $t + \delta t$, which satisfies $\pi_{ij} \ge 0$, for $j \ne i$ and $\sum_{j=1}^{N}\pi_{ij} = 0$. To simplify the notation, $A^{(i)}$ and $B^{(i)}$ denote the $A(r(t) = i)$ and $B(r(t) = i)$, respectively. Further, for the vector $\sigma = \begin{bmatrix}\sigma_1 & \cdots & \sigma_m\end{bmatrix}^T \in \mathbb{R}^m$, the saturation operator $sat(\cdot)$ is defined as

$$[sat(\sigma)]_k \triangleq \begin{cases} 1 & \sigma_k \ge 1, \\ \sigma_k & |\sigma_k| < 1, \\ -1 & \sigma_k \le -1, \end{cases} \tag{3}$$

where $[sat(\sigma)]_k$ is k_{th} element of $sat(\sigma)$.

Definition 1. System (1) with $u(t) \equiv 0$ is said to be positive if for any initial condition $x_0 \succeq 0$, the corresponding trajectory $x(t) \succeq 0$ holds for all $t > 0$.

Lemma 1 (*[25]*)**.** *System (1) with $u(t) \equiv 0$ is positive if and only if $A^{(i)}$ is a Metzler matrix.*

Lemma 2 (*Cao et al. [26] and Hu and Lin [27]*)**.** *Let $u, u^v \in \mathbb{R}^m$,*

$$u = [u_1 \;\, u_2 \;\, \cdots \;\, u_m]^T, \quad u^v = \left[u_1^v \;\, u_2^v \;\, \cdots \;\, u_m^v\right]^T. \tag{4}$$

Assume that $|e_i^T u^v| \le 1$ for all $i \in \mathbb{N}_m^+$, then

$$sat\,(u) \in \mathbf{Co}\left\{\, D_k u + D_k^- u^v \quad k \in \mathbb{N}_{2^m}^+ \,\right\}, \tag{5}$$

*where e_i is a unit vector with the i_{th} nonzero entry, i.e., $e_i \triangleq [0 \cdots \underbrace{1}_{i_{th}} \cdots 0]^T$, D_k denotes a diagonal matrix with all possible combinations of 1 and 0 diagonal entries, $D_k^- \triangleq I - D_k$, and **Co** is the convex hull.*