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State bounding for switched homogeneous positive nonlinear systems with exogenous input

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ABSTRACT

This paper considers the state bounding problem for switched homogeneous positive nonlinear systems with bounded exogenous input. Both continuous-time switched positive nonlinear systems whose vector fields are homogeneous of degree one and cooperative, and discrete-time switched positive nonlinear systems whose vector fields are homogeneous of degree one and order preserving are discussed under average dwell time switching. By using a new method, we derive necessary and sufficient conditions for the existence of a ball where all the solutions of the system converge exponentially within. Numerical examples are also presented to illustrate the theoretical results.

1. Introduction

Positive dynamical system is one for which non-negative initial conditions give rise to non-negative trajectories [1]. Positive systems model many physical systems, for example population sizes, commodity prices and chemical concentrations. In the recent years, the analysis of positive systems has attracted significant attention due to their importance.

Many important positive systems are nonlinear, and the stability theory of positive nonlinear systems has been comprehensively studied by researchers. Homogeneous cooperative system belongs to a particular class of positive nonlinear systems [2–4]. For stability of homogeneous cooperative systems, Feyzmahdavian et al. studied the exponential stability of homogeneous cooperative systems of degree one with bounded time-varying delay in [5], and then extended the results to the more general homogeneous cooperative system in [6]. Dong discussed the decay rate of homogeneous positive systems of any degree with time-varying delays in [7].

Switched systems consist of a finite number of subsystems and a switching signal specifying a subsystem to be active during an interval of time [8], which have been highlighted by many researchers due to their extensive applications in engineering. The stability analysis of switched systems is also a fundamental research issue [9–24]. Under arbitrary switching signals, many switched systems may fail to preserve stability. Therefore, the average dwell time (ADT) switching is always considered [25–31].

In practice, some positive systems may have nonnegative exogenous input such as the external voltage in the power system. In the presence of exogenous input, we are interested in the existence of a neighborhood of the equilibrium point where the solutions of the system converge exponentially within. The problem of state bounding for linear time-varying systems with bounded disturbance was studied in [32]. Recently, we considered the state bounding problem for

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homogeneous positive nonlinear systems of degree one with bounded nonnegative exogenous input in [33], and gave necessary and sufficient conditions which guarantee that all the solutions of the system converge exponentially within a ball. For the homogeneous positive nonlinear system of degree greater than one, a necessary and sufficient condition for state bounding of the system was established in [34].

To the best of our knowledge, under ADT switching no result has been obtained on the problem of state bounding for the switched homogeneous positive nonlinear systems of degree one with nonnegative exogenous input. Inspired by this and some related work in the literature, in this paper we intend to derive necessary and sufficient conditions guaranteeing that all the solutions of the switched homogeneous positive nonlinear systems of degree one under ADT switching converge exponentially within a ball for both the continuous-time case and the discrete-time case, which complement the results presented in [33] and [30] to some extent.

The remainder of this paper is organized as follows. Section 2 presents notations and some necessary preliminaries. Section 3 states the main results of this work. Section 4 gives numerical examples to illustrate the obtained results. Section 5 concludes this paper.

2. Problem description and preliminaries

Throughout this paper, R, N and N_0 denote the set of real numbers, natural numbers, and natural numbers including zero, respectively. $\mathbb{R}^n$ is the n-dimensional real space and $R^n_+ = \{x = (x_i) \in R^n, x_i \geq 0, 1 \leq i \leq n\}$. For $x = (x_i) \in R^n, y = (y_i) \in R^n$, denote $x \geq y$ ($x \gg y, x \ll y$) if $x_i \geq y_i$ ($x_i > y_i, x_i < y_i$) for $1 \leq i \leq n$. Given a vector $v \in R^n$, $v \gg 0$, the weighted l_∞ norm is defined by $\|x\|^v_\infty = \max_{1\leq i\leq n} \frac{|x_i|}{v_i}$. Set $\|x\|_\infty = \max_{1\leq i\leq n}|x_i|$. Given $\varepsilon \in R$ and $\varepsilon > 0$, let the ball $\mathcal{B}(\varepsilon)$ be defined by $\mathcal{B}(\varepsilon) := \{x \in R^n, \|x\|_\infty \leq \varepsilon\}$. Let $R^{n\times n}$ denote the space of $n \times n$ matrices with real entries. For matrix $A \in R^{n\times n}$, a_{ij} denotes the (i, j)th entry of A. A matrix $A \in R^{n\times n}$ is said to be Metzler if $a_{ij} \geq 0$ for all $i \neq j$.

Definition 1 (*[3]*). A continuous vector field $f : R^n \to R^n$, which is C^1 on $R^n \setminus \{0\}$, is said to be cooperative if the Jacobian matrix $(\partial f/\partial x)(a)$ is Metzler for all $a \in R^n_+ \setminus \{0\}$.

Cooperative vector fields satisfy the following property.

Proposition 1 (*[2]*). *Let vector field $f : R^n \to R^n$ be cooperative. For any two vectors $x, y \in R^n_+$, with $x_i = y_i$, and $x \geq y$, we have $f_i(x) \geq f_i(y)$.*

Definition 2 (*[5]*). $f : R^n \to R^n$ is said to be homogeneous of degree α if for all $x \in R^n$ and all real $\lambda > 0, f(\lambda x) = \lambda^\alpha f(x)$.

When $\alpha = 1, f$ is called homogeneous of degree one.

Definition 3 (*[5]*). $f : R^n \to R^n$ is order-preserving on R^n_+ if $f(x) \geq f(y)$ for any $x, y \in R^n_+$ such that $x \geq y$.

In this paper, we consider both the continuous-time and discrete-time switched nonlinear systems with exogenous input. The continuous-time switched nonlinear system takes the form

$$\dot{x}(t) = f_{\sigma(t)}(x(t)) + w(t), \quad t \geq 0, \tag{2.1}$$

where $x(t) \in R^n$ is the state vector; the switching signal $\sigma(t) : [0, +\infty) \mapsto \mathcal{P}$ is a piecewise constant, right continuous function, which takes its values in the finite set $\mathcal{P} = \{1, \ldots, M\}$, M is the number of subsystems; $f_p : R^n \to R^n$ are vector fields with $f(0) = 0$; $w(t) : [0, \infty) \to R^n$ is the exogenous input vector. Denote the switching times by $0 = t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots$. When $t \in [t_k, t_{k+1})$, the $\sigma(t_k)$th subsystem is active.

The discrete-time switched nonlinear system considered in this paper takes the form

$$x(k+1) = f_{\sigma(k)}(x(k)) + w(k), \quad k \in N_0, \tag{2.2}$$

where $x(k) \in R^n$ is the state vector; the switching signal $\sigma(k) : N_0 \mapsto \mathcal{P}$; $\mathcal{P}$ and f_p are defined above; $w(k) : N_0 \to R^n$ is the exogenous input vector. Denote the switching times by $0 = k_0 < k_1 < \cdots < k_l < k_{l+1} < \cdots$. When $k \in [k_l, k_{l+1})$, the $\sigma(k_l)$th subsystem is active.

Definition 4. The solution $x(t)$ of system (2.1) is said to converge exponentially within a ball if there exist constants $a \geq 0$, $b > 0$ and $\lambda > 0$ such that $\|x(t)\|_\infty \leq a + be^{-\lambda t}$ for $t \geq 0$, where the constant a depends on the upper bound $\bar{w}$ of $w(t)$ and $a = 0$ when $\bar{w} = 0$, the constant b is related to the initial condition $x(0)$ and $\bar{w}$.

Definition 5. The solution $x(t)$ of system (2.2) is said to converge exponentially within a ball if there exist constants $a \geq 0$, $b > 0$ and $0 < \gamma < 1$ such that $\|x(k)\|_\infty \leq a + b\gamma^{-k}$ for $k \in N_0$, where the constant a depends on the upper bound $\bar{w}$ of $w(k)$ and $a = 0$ when $\bar{w} = 0$, the constant b is related to the initial condition $x(0)$ and $\bar{w}$.