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Disturbance decoupling in nonlinear hybrid systems

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ABSTRACT

The paper studies the problem of disturbance decoupling of nonlinear hybrid systems. The hybrid systems under consideration are switched systems that consist of finite automaton, which defines the switching rule, a set of nonlinear discrete-time systems and the so-called mode activator, that defines an input for the automaton. Such systems allow to handle more complex switching rules than just time- or state-dependent switchings. The goal of the paper is to achieve the disturbance decoupling by dynamic measurement feedback. The advantage of such feedback is that it does not require estimations of state variables. Sufficient conditions are found under which there exists a dynamic measurement feedback, such that in the closed-loop system the controlled output of the hybrid system does not depend on the disturbance. An algebraic approach called functions' algebra is used, which can address in a similar manner both discrete-time systems and discrete-event systems (finite automaton).

1. Introduction

Hybrid systems represent a large and important class of systems, which arise when different technological or mathematical objects are combined [1]. For example, systems that contain analog and digital components or systems where some parts are described by automaton, other parts by differential equations. A typical example of hybrid system is a switched system, which consists of several subsystems and a switching rule, that indicates the active mode among the subsystems. The switching can depend on the time or on the values of the states, it may be autonomous or controlled. Switched systems have various applications in the fields like flight control [2], engine control [3], network control system [4] etc.

Most often the switching rule is defined by a function from the time domain to the set of system modes (when the switching depends on the time) or by a function from the state space to the set of system modes (when the switching depends on the state variables). However, the book [5] suggests that the decision-making component of the hybrid system can be chosen as a finite automaton, or as a more general discrete-event system. This, of course, contains also the cases when switching depends only on time or state variables. By looking the switching as finite automaton, one can construct more advanced switching rules. For example, a car can be viewed as a hybrid system, since its dynamics depends on the selected gear. Now, finite automaton allows to define switching (i.e., gear changes), such that gears are changed only by one up or down, which is a realistic situation. In principle, it means that the switching depends not only on the system states, but also on the system modes.

In this paper we consider hybrid systems, whose behavior is determined by interaction of a number of discrete-time nonlinear systems (difference equations) and a finite automaton (discrete-event dynamics). The discrete-time system depends on parameters that take values from a finite set of real numbers and the values of the parameters are determined

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by the outputs of the finite automaton. The switching occurs whenever the output of the finite automaton changes. These changes can happen due to the dynamics of the finite automaton or because the input of the finite automaton changes. The latter happens when the state moves from one predefined domain of the state space to the other. In the special case when there is no dynamics of finite automaton, the switching depends solely on the state variables. This type of hybrid systems has been studied earlier in [6,7] with respect to reduced order observer design and fault diagnosis problems, respectively.

In this paper, the problem of dynamic disturbance decoupling via measurement feedback (DDDPM) is studied for hybrid systems. The problem of disturbance decoupling has been studied earlier for switched linear systems without delays [8–12] and with time delays [13], and also for switched nonlinear systems [14]. Except [12], all these papers use state feedback to achieve the decoupling. However an output/measurement feedback would be preferable, since it does not require the knowledge of state variables. In this paper, the goal is to find a dynamic measurement feedback, such that the functions describing the controlled output of the closed-loop system do not depend on the disturbance at any time instant. The feedback itself has similar structure as the hybrid system. For each nonlinear discrete-time mode, one has to construct a measurement feedback that solves the DDDPM for that mode (this problem has been solved in [15]). Then, one has to construct a switching rule for the controllers. Note that it is not enough to solve the DDDPM for each mode, since the disturbance can affect the system output through the switching rule, which may be affected by the disturbance.

Besides solving the disturbance decoupling problem, it is important to guarantee that the closed-loop system is stable. Some contributions solve the two problems as one [10–12]. Stability issues for the class of hybrid systems, considered in this paper, are not studied before nor is the algebraic approach, employed in this paper, useful for dealing with stability issues. Stability of discrete-time nonlinear switched systems is not much studied [16] and most contributions for the other system classes consider time-dependent switching rule. However, it is well-known that the existence of a common Lyapunov function that satisfies certain conditions is sufficient for the stability of the switched system under arbitrary switching signals, see [16] and references therein. To guarantee stability of the closed-loop system, the results of this paper can be combined with the results from [16] on stability under arbitrary switching signals.

The algebraic tools that will be applied (algebra of partitions/functions) work both for the discrete-time systems and the finite state automaton and therefore suit well to study the hybrid systems. The algebra of partitions was developed for the finite automaton in [17]. Similar approach for the discrete-time systems, called algebra of functions, see [18,19], was inspired by the approach in [17] and mimics the latter. Since the two theories are interlinked, it helps to build a bridge between the theories of finite automaton and the discrete-time systems. The partitions were replaced by the functions, generating them and the analogous operations/operators were introduced.

The preliminary results of this paper were presented in the conference article [20]. Here, the results of [20] have been improved considerably. First, in the problem statement of [20] the switching rule of the compensator was allowed to depend on the measured output y, which actually is forbidden, because it may depend on the disturbance. Current paper fixes this error by restating and proving the sufficient conditions (Theorem 2 in this paper). Second, the sufficient conditions (when compared to those in [20]) are relaxed. Third, we comment here the more general case when disturbance affects also directly the finite automaton. The results of the paper have strong connection to the papers [15,21], that solve the DDDPM for nonlinear discrete-time systems. As different modes in our hybrid system are discrete-time systems, the solution relies naturally on these papers. However, solving only the DDDPM for different modes is not enough to guarantee solvability for the hybrid system. In summary, the novel aspects of the paper are: (1) use of *more general switching rule* in the study of switched systems, (2) use of *measurement feedback* in the study of nonlinear switched systems and, (3) *sufficient conditions* for the existence of a dynamic measurement feedback, that solves the DDDPM for a given hybrid system.

The rest of the paper is organized as follows. In Section 2, the hybrid system is described and the problem statement is formulated. Section 3 recalls basic notions from the algebra of partitions/functions, important to understand the results of this paper. Section 4 presents the main results of the paper. First, the solution of the DDDPM for the discrete-time systems is recalled. Then the DDDPM is solved for the hybrid system and the construction of the compensator is described. Finally, some words are said about a more general problem statement, where the disturbance affects directly also the switching rule, i.e. the finite automaton. Section 5 presents two examples, which is followed by the conclusion.

2. Basic models and problem statement

The hybrid system (HS) studied in this paper is represented schematically in Fig. 1. In this figure we see that there are three basic elements: the finite automaton (FA), the discrete-time system (DTS) (actually a set of them, one for each mode), and the so-called mode activator (MA), that coordinates the actions of the FA and the DTS. The FA is described by the model

$$A = (I, S, O, \delta, \lambda),$$

where I, S, and O are finite sets of inputs, states and outputs, respectively. The state transition and output functions of the FA are described by

$$\begin{aligned} s^+ &= \delta(s, i), \\ o &= \lambda(s), \end{aligned} \tag{1}$$

where $s^+ \in S$ is the new state after transition from the state $s \in S$, initiated by the input $i \in I$, and $o \in O$ is the output. It is assumed that both functions δ and λ are specified by the appropriate tables. The automaton A is assumed to be minimal (irreducible), i.e. the number of its states cannot be decreased, see [22] for definition of minimal automaton.