Nonlinear Analysis: Hybrid Systems 28 (2018) 54–74

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Periodic solutions of a two-degree-of-freedom autonomous vibro-impact oscillator with sticking phases

Huong Le Thi [a,b], Stéphane Junca [a,*], Mathias Legrand [c]

[a] *Université Côte d'Azur, LJAD, CNRS & Inria, Nice, France*
[b] *Department of Mathematics and Informatics, Thang Long University, Nghiem Xuan Yem Road, Hoang Mai District, Ha Noi City, Viet Nam*
[c] *Department of Mechanical Engineering, McGill University, Montréal, Québec, Canada*

ARTICLE INFO

Article history:
Received 25 March 2016
Accepted 25 October 2017
Available online 4 January 2018

Keywords:
Vibration
Impact
Sticking phase
Periodic solutions
2 dof

ABSTRACT

This contribution explores the free dynamics of a simple two-degree-of-freedom vibro-impact oscillator. One degree-of-freedom is unilaterally constrained by the presence of a rigid obstacle and periodic solutions involving one sticking phase per period (1-SPP) are targeted. A solution method to obtain such orbits is proposed: it provides conditions on the existence of 1-SPP as well as closed-form solutions. It is shown that 1-SPP might not exist for a given combination of masses and stiffnesses. The set of 1-SPP is at most a countable set of isolated periodic orbits. The construction of 1-SPP requires numerical developments that are illustrated on a few relevant examples. Comparison with nonlinear modes of vibration involving one impact per period (1-IPP) is also considered. Interestingly, an equivalence between 1-SPP and a special set of isolated 1-IPP is established. It is also demonstrated that the prestressed system features sticking phases of infinite duration.

1. Introduction

In the context of vibration and modal analysis of vibro-impact oscillators, nonlinear modes of vibration with non-grazing impact are explored in [1–6]. In these works, a single degree-of-freedom (dof) is unilaterally constrained by the presence of a rigid foundation: the dynamics is purely linear when the contact constraint is not active, and governed by an impact law otherwise. Accordingly, the contact force arising when the system interacts with the foundation is a periodic distribution of Dirac deltas.

Instead, the present work pays attention to periodic solutions involving long "sticking" contact phases, thus discarding impulse-driven dynamics reported in the previous works, during which the contacting mass rests against the obstacle for a finite amount of time. Let us clarify the terminology now: as explained later, "sticking" here means that the impacting mass will rest on the rigid foundation for a finite or infinite amount of time during its motion even though there is no "adhesion force" arising from the rigid foundation and acting on the impacting mass. An equivalent terminology would be "lasting non-impulsive" contact phases. This investigation is motivated by the fact that in a continuous setting in space and time, unilateral contact forces are known to be discontinuous functions at most [7] while they become impulsive after a semi-discretization in space via the Finite Element technique. The question is then: are there non-impulsive solutions in the Finite Element framework and alike?

* Corresponding author.
E-mail addresses: lethih@unice.fr (H. Le Thi), junca@unice.fr (S. Junca), mathias.legrand@mcgill.ca (M. Legrand).

https://doi.org/10.1016/j.nahs.2017.10.009
1751-570X/

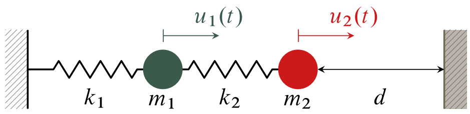

Fig. 1. Two degree-of-freedom vibro-impact system at equilibrium, $d > 0$.

Solutions having a sticking phase are a specific feature of vibro-impact oscillators. The grazing contact gives a non smooth dynamics with singular phenomena in [8,9]. Notice also that periodic solutions with a sticking phase has also been recently investigated in [10] for N dof with another approach.

The system of interest is an oscillator with two masses m_1 and m_2 linearly connected through two springs of stiffness k_1 and k_2 respectively, as depicted in Fig. 1. The dynamics of interest reads:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{r} \tag{1.1a}$$

$$\mathbf{u}(0) = \mathbf{u}_0, \quad \dot{\mathbf{u}}(0) = \dot{\mathbf{u}}_0 \tag{1.1b}$$

$$u_2(t) \le d, \quad R(t) \le 0, \quad (u_2(t) - d)R(t) = 0, \quad \forall t \tag{1.1c}$$

$$\dot{\mathbf{u}}^\top \mathbf{M}\dot{\mathbf{u}} + \mathbf{u}^\top \mathbf{K}\mathbf{u} = \mathbf{E}(t) = \mathbf{E}(0) \tag{1.1d}$$

where

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}, \quad \mathbf{u}(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} \quad \text{and} \quad \mathbf{r}(t) = \begin{pmatrix} 0 \\ R(t) \end{pmatrix}.$$

Above, u_j, $\dot{u}_j$, and $\ddot{u}_j$ represent the displacement, velocity, and acceleration of mass j, $j = 1, 2$, respectively. The fixed gap d is defined between the obstacle and the equilibrium position of the second mass. It is the algebraic *distance* between mass 2 at rest without any external force and the rigid wall and might thus be negative in the prestressed case. The quantity $R(t)$ is the reaction force of the wall on mass 2.

Matrices $\mathbf{M}$ and $\mathbf{K}$ are symmetric positive definite so that there is a matrix $\mathbf{P}$ of eigenmodes which simultaneously diagonalizes both of them, that is $\mathbf{P}^\top\mathbf{M}\mathbf{P} = \mathbf{I}$ and $\mathbf{P}^\top\mathbf{K}\mathbf{P} = \boldsymbol{\Omega}^2 = \mathbf{diag}(\omega_i^2)|_{i=1,2}$ where $\mathbf{I}$ is the 2×2 identity matrix and ω_i^2, $i = 1, 2$ are the eigenfrequencies of the linear system without unilateral contact.

Eq. (1.1d) reflects the conservative nature of the system. Energy is preserved along a motion. Eq. (1.1d) implies the existence of a perfectly elastic impact law of the form $\dot{u}_2^+ = -e\dot{u}_2^-$ with $e = 1$ where $\dot{u}_2^-$ and $\dot{u}_2^+$ respectively stand for the pre- and post-impact velocities of mass 2. As detailed later, 1-SPP orbits are defined to be independent of the restitution coefficient e. Thus 1-SPP still exist for $e \in [0\,;1]$. However, the framework $e = 1$ is chosen for comparison with another class of periodic solutions with nonzero velocity at the impact. Moreover, the structure of a general solution is simpler when $e = 1$. This initial-value problem with conserved energy is known to be well-posed [11,12]. The sticking phase is known to appear as a limit of a chattering sequence [8,13–15]. However in the present work, there is no source term and sticking periodic solutions can still occur.

The paper is organized as follows. In Section 2, the definition of a “sticking phase” is provided and conditions for its occurrence are stated. Then, necessary conditions satisfied by periodic solutions with one sticking phase per period (1-SPP) are given in terms of the free flight duration s. The parameter s appears to be the key parameter and a root of an explicit nonlinear function. Furthermore, when s is known, the corresponding 1-SPP is expressed in closed form. The method and numerical examples are described in Section 3 in order to find all 1-SPP.

Mathematical proofs are detailed in Sections 4–6. More precisely, Section 4 deals with the structure of the space of solutions with a sticking phase. Section 5 is devoted to prove Theorem 2.1 on 1-SPP. The existence result of an infinite set of admissible initial data such that the associated solutions satisfy the constraint $u_2 \le d$ near the sticking phase is proven in Section 6. The existence of 1-SPP satisfying the constraint $u_2 \le d$ during the whole period remains an open problem. The prestressed case with $d \le 0$ is discussed in Section 7. Section 8 concludes the paper. In this work, all numerical simulations are performed using the parameters $m_1 = m_2 = 1$ and $k_1 = k_2 = 1$, unless indicated otherwise.

2. Main results

The occurrence of a sticking phase is first defined with necessary and sufficient conditions in Section 2.1. Then, in Section 2.2, periodic solutions with one sticking phase per period, the so-called 1-SPP, are characterized through necessary conditions to exhibit them all. Throughout the current work, internal resonances are discarded, i.e. $\cap_{j=1}^{2} T_j\mathbb{N} = \emptyset$, unless stated otherwise.

2.1. Sticking phase

A sticking phase occurs when the mass number 2 stays at $u_2 = d$ on a proper time interval.