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Reduced-order approximation of consensus dynamics in networks with a hybrid adaptive communication protocol

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ABSTRACT

This paper focuses on a hybrid adaptive control (or communication protocol) that is used to drive a network of agents towards consensus. We develop an approximation to the system to capture the qualitative behaviour of the most distant node state and gain dynamics, thus serving as a powerful reduction of order of the larger system. Numerical results are presented to argue the robustness of the approximation over a range of differing initial network sizes and distributions of initial state values.

1. Introduction

The study of multi-agent systems in the context of dynamic networks has wide applications in areas as diverse as neuroscience [1], systems biology [2], social networks [3,4], unmanned air-vehicle control [5], guidance systems [6] and distributed sensor networks [7].

One important emergent property of multi-agent systems that we may wish to control is that of *consensus* [8–11]. Broadly speaking, consensus occurs when some agreement is reached via the sharing of information between the agents in a network. Flocking behaviour in animals [12] or formation control of autonomous vehicles [13] can be viewed broadly as different types of consensus since both rely on information sharing and adaptation within the ensemble in order to bring the larger group into unison or some establish some favourable configuration [10,13–15].

To direct a network towards consensus some *communication protocol* is typically employed by the agents in the network, where said protocol can either be *centralised*, *decentralised* or a combination of the two. Crucially, in centralised control strategies each agent is externally controlled in order to achieve a preset configuration, whereas with decentralised control protocols no ultimate configuration is prescribed from the outset. In the latter strategy the agents are allowed to communicate with one another and adapt naturally in response to changes in their neighbours' states, effectively self-organising the network until the ensemble reaches consensus [5,16,17].

The strength of the communicative relationships between respective agents in an ensemble might be quantified in some way and allowed to evolve over time. Should the weight of these interactions represent, for example, the quantity of information relayed between people, there may come a point when the amount of information received is sufficient to convey a message. Once this message has been received, it would be redundant to continue to increase the information flow any further beyond this threshold. It is natural in this scenario to imagine a *hybrid* communication protocol that permits the influences to grow up to some critical threshold and cease evolving thereafter.

The goal of the present paper is to focus on a particular hybrid control strategy which we call the *switch protocol* and formulate an approximation that dramatically reduces the order of the corresponding large-scale dynamical system,

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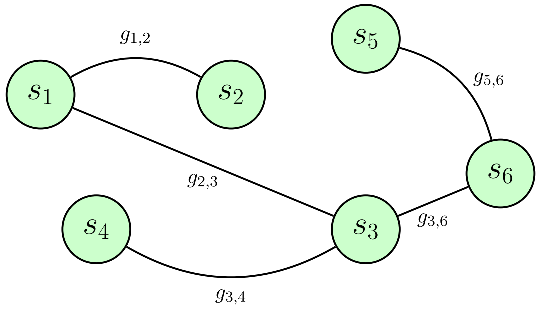

Fig. 1. Schematic of a six-node network with weighted edges between some of the nodes.

allowing us to track an arbitrarily large system with a computationally inexpensive reference system of three ordinary differential equations.

The outline of the paper is as follows. Section 2.1 will introduce the nomenclature and some of the key concepts that shall be relied upon throughout the rest of the paper. Section 2.2 defines the adaptive control and the particular mathematical model under investigation. Section 2.3 identifies some salient features of the model and motivates a need for an approximation to the complex dynamics. Section 3 details the construction of the reduced-order approximation whilst Section 4 presents numerical results to show its efficacy. Finally, a short discussion and some open questions are considered in Section 5.

2. Network dynamics

2.1. Evolution and consensus

A *network* is a weighted graph that is made from a set of elements called *nodes* or *vertices*, which may be connected to one another via relational links called *edges* or *arcs*. More formally, the network topology of a finite system of N agents $G = (V, E)$ comprises a set of node elements $V \in \mathbb{R}^N$ and a set of arcs $E \subset V \times V$. The set of nodes is given by

$$V := \{v_i : i = 1, \dots, N\}$$

with each v_i describing one of the N nodes while the set of arcs E catalogs the existing links between each pair of nodes. We restrict our attention to undirected symmetric graphs (as opposed to digraphs) hence no distinction is made between $e_{i,j}$ and $e_{j,i}$, the edge that links v_i to v_j. Furthermore we do not allow self loops hence

$$E := \{e_{i,j} : i = 1, \dots, N,\ j = 1, \dots, N,\ i \neq j\}.$$

To each node v_i we assign an instantaneous *state* value $s_i = s_i(t) \in \mathbb{R}$ and to each edge $e_{i,j}$ an instantaneous weight or *gain* $g_{i,j} = g_{i,j}(t) \geq 0$. Fig. 1 shows a six-node network, where every node is given a state value and each existing arc is assigned a weight. For good introductions to a variety of different types of networks see [1] and [18].

Given a network of N identical nodes representing a dynamical system of interacting agents, we are interested in the evolution of this system, i.e. the state and gain dynamics over time. Here, we will consider an *edge-based* network evolution whereby the gains grow (or remain constant) as the agents communicate with one another, reorganising and changing their state values over time.

In the context of network models, with node states s_i, this is a *consensus* problem inasmuch as the aim is to drive all of the node states towards a common value. We define a consensus in the system to have occurred when, for every pair of node states s_i and s_j, we have

$$\lim_{t \to \infty} (s_i - s_j) = 0, \tag{1}$$

where t is time. As we shall see in Section 4, in practice, consensus may be said to have occurred if all pairs of state differences lie within some small numerical tolerance $0 < \epsilon \ll 1$. The key to achieving consensus is an appropriate communication protocol between the network nodes. This protocol is the mechanism by which the agents relay information to and from their neighbours and adapt their states accordingly. As we shall see, the communication protocol will be likewise driven by a function of the state differences. For the sake of convenience we define the difference between a given pair of node states as

$$\Delta_{i,j} = s_i - s_j, \tag{2}$$