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# Switching and impulsive control algorithms for nonlinear hybrid dynamical systems

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ABSTRACT

Control algorithms are developed for physical processes modeled as hybrid dynamical systems (HDSs). In this framework, a HDS is a nonlinear switched system of ordinary differential equations (ODEs) coupled with impulsive equations. Switching and impulsive control is applied with two performance goals in mind: First, a high-frequency switching control method is provided to drive a HDS state to the origin while only requiring the HDS state intermittently. Attractivity of the origin is proved under a shell bisection algorithm; a high-frequency switching control rule is designed for this purpose. Second, a state-dependent switching control strategy is derived for when the transient behavior of the HDS is of interest. Finite-time stabilization is guaranteed under a so-called minimum rule algorithm; for each HDS mode, the state space is divided into different control regions and a switching control rule is constructed to switch between controllers whenever a boundary is reached. The theoretical tools used in this article include the Campbell–Baker–Hausdorff formula, multiple Lyapunov functions, and average dwell-time conditions.

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## 1. Introduction

Hybrid dynamical systems (HDSs) have recently gained much research attention as they provide a natural modeling framework for physical processes in many areas of engineering (see, e.g., [1–8] and the references therein). Practical examples can be found in robotics, mechanical systems, the automotive industry, air traffic control, intelligent vehicle/highway systems, chaos generators, among others [7,9–11]. An HDS modeled according to a mixture of continuous/discrete dynamics and logic-based switching, which is the focus in this article, usually arises in two contexts [4,9]: a physical process whose governing dynamics change abruptly in time; and a dynamical system being stabilized via switching controllers. There are a variety of reasons why switching control is desirable, or even required, over a continuous control strategy [1,7,9]: it may not be possible to implement or even find a continuous control because of the problem's nature, uncertainty in the model, or sensor/actuator limitations; performance of the control strategy can be improved under switching control; and switching control may be easier to develop or the only viable option (e.g., in stabilizing an unstable system in which no continuous control exists). This serves as the main motivation for the present article which focuses on the design of switching control algorithms for complex physical processes, modeled by HDSs, to achieve desired performance behaviors.

Switched systems can exhibit unintuitive stability behavior; a HDS composed entirely of stable modes can display instability while one composed entirely of unstable modes can demonstrate stability [1]. This latter observation motivates open-loop high-frequency and closed-loop state-dependent switching control stabilization strategies. The open-loop switching control scheme involves the construction of a high-frequency stabilizing switching rule, constructed *a priori*; the control

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strategy information (i.e., the switching control times and mode sequence) is preprogrammed into the data. The authors Bacciotti and Mazzi [12] studied nonlinear switched systems and proved the existence of a solution to said open-loop problem. Stabilization of nonlinear systems to a compact set using a time-dependent switching rule was considered by Mancilla-Aguilar and Garcia in [13], and other such investigations can be found in, for example, [14–16]. On the other hand, the closed-loop state-dependent switching approach, first developed by Wicks et al. [17] to stabilize an unstable linear system via switching control, has been studied more extensively in the literature. The main idea is as follows: given an unstable system and a set of feedback controllers (none of which succeed individually in stabilizing the system), the state space is partitioned into different regions and an algorithm is devised in order to dictate a control switching rule for choosing the active feedback controller in each such region. This line of research has been extended in a number of directions, such as nonlinear switched systems [18], linear systems with time-delays [19], systems with distributed delays (see, e.g., [20–23]), and state-dependent switching and impulsive control [24–26].

A significant portion of the current switched systems stability literature focuses on Lyapunov asymptotic stability, defined over an infinite time horizon. However, in many practical applications in engineering fields related to systems and control analysis, the behavior of the transient over a finite time horizon is of paramount importance; for example, in guaranteeing that system states remain in a safe operating range and avoid violating safety operating conditions [27–29]. Finite-time stability (sometimes called short-time stability) was first introduced and developed in [30,31] and can be stated as follows: given a bound on the initial state, determine whether the state trajectory remains within a prescribed bound in a fixed time horizon. Unfortunately, these early results were not practical from an analysis and control synthesis viewpoint [29]. Recently, the theory of linear matrix inequalities (LMIs) has been used to revisit the finite-time stability problem [32], allowing for less conservative conditions under which finite-time stabilization can be achieved [29]. The authors Amato et al. [29] proved finite-time stability of quadratic systems under prescribed bounds involving polytopes by using LMIs and feasibility problems in convex optimization. Amato et al. [32] investigated finite-time boundedness of linear systems subject to parametric uncertainties and exogenous disturbances. Finite-time stabilization of discrete-time systems subject to disturbances was considered using LMIs by Amato and Ariola [33]. The authors Amato et al. [28] used LMIs to extend the finite-time stabilization literature by using a dynamic output feedback controller (rather than a state feedback controller). In [27], Du et al. presented sufficient conditions for finite-time stability of linear switched systems using LMIs, average dwell-time notions, and multiple Lyapunov functions. Du et al. [34] analyzed finite-time boundedness and stabilization of switched linear systems with disturbances using a state-dependent switching strategy and multiple Lyapunov functions. Other investigations can be found in, for example, [35–37].

The objective of this paper is to develop control algorithms for a general nonlinear HDS. Motivated by the above discussions, a major area of research of switched systems is thus extended by detailing how switching control and impulsive control can be used together to successfully realize certain desired performances. In this formulation, hybrid systems are studied which have independent switching rules: a logic-based rule driven by factors associated with the uncontrolled HDS and a control rule designed according to an algorithm. First, an infinite time horizon problem is considered in which sensors are available intermittently at the HDS switching times. A new method, called the exponential shell bisection algorithm, is developed which explicitly constructs an open-loop high-frequency control switching rule for a HDS. This extension of [12] may be viewed as a type of output feedback control (where the output is the full system state, but only available intermittently at the HDS switching times). This algorithm ensures attractivity (local or global), which has applications in synchronization problems (see, e.g., [38–41]). When the transient behavior of the physical system is of interest, a closed-loop state-dependent switching algorithm is implemented which guarantees finite-time stabilization. The state-dependent switching control work in the present article broadens the current works detailed above by showing finite-time stabilization of a nonlinear HDS exhibiting average dwell-time condition via multiple Lyapunov functions (see, e.g., [1,42,43]). New stability results are found for autonomous dwell-time switching and average dwell-time switching, extending the results found in [10,44].

The rest of the article is organized as follows: in Section 2, the hybrid process control problem is outlined after some preliminaries are given. A high-frequency hybrid control algorithm is presented and studied in Section 3. A finite time horizon refinement of the problem is detailed in Section 4, where a finite-time stabilization algorithm is outlined using state-dependent switching control. Examples are provided throughout to illustrate the theoretical results. Conclusions and possible future work are given in Section 5.

## 2. Problem formulation

The following notations are used throughout this paper: Let $\mathbb{R}_+$ denote the set of nonnegative real numbers and $\mathbb{R}^n$ denote the Euclidean space of $n$-dimensions (equipped with the Euclidean norm $\|\cdot\|$). Let $\mathbb{Z}$ denote the set of integers and $\mathbb{Z}_{>0}, \mathbb{Z}_{\geq 0}, \mathbb{Z}_{\leq 0}, \mathbb{Z}_{<0}$ denote the set of positive, nonnegative, nonpositive, and negative integers, respectively. Let $\mathbb{R}^{n\times m}$ denote the set of $n \times m$ matrices equipped with the corresponding induced norm. Given a symmetric matrix $Q \in \mathbb{R}^{n\times n}$, let $\lambda_{\max}(Q)$ and $\lambda_{\min}(Q)$ denote its maximum and minimum eigenvalue, respectively. Let $B(x, r)$ denote the open ball of radius $r > 0$ centered at $x \in \mathbb{R}^n$. Given a set $A \subset \mathbb{R}^n$, let $\mathrm{cl}(A)$ denote the closure of $A$ and let $\partial A$ denote the boundary of $A$. Let $\delta(\cdot)$ denote the generalized Dirac delta function and let $\lceil\cdot\rceil$ denote the ceiling function. Let $\mathcal{H}(\mathbb{R}^n, \mathbb{R}^m)$ denote the space of analytic functions mapping $\mathbb{R}^n$ to $\mathbb{R}^m$ (equipped with an appropriate norm so that it is a Banach space), and let $\mathcal{H}_r := \mathcal{H}(\mathbb{R}^n, \mathbb{R}^n)$ denote the space of analytic functions bounded on $B(0, r) \subset \mathbb{R}^n$ for some $r > 0$. Let $C(\mathbb{R}^n, \mathbb{R}^m)$, and $C^1(\mathbb{R}^n, \mathbb{R}^m)$ denote the spaces of

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