



Ergodic property of the chemostat: A stochastic model under regime switching and with general response function

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HIGHLIGHTS

- Stochastic chemostat under regime switching with general growth rate is developed.
- The stochastic model has a unique stationary distribution which is ergodic.
- Sufficient conditions for ergodicity are in terms of white noise and Markov chain.
- The conclusions are valid for many prototypes of response functions.

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ABSTRACT

The dynamics of chemostat involving stochastic perturbations is considered. Instead of assuming the familiar Monod kinetics for nutrient uptake, a general class of functions is used which includes all monotone (non-monotone) increasing uptake functions. We make two classifications of restrictive assumptions on the growth response. For the chemostat disturbed by both white noise and colored noise, we obtain the ergodic property by constructing stochastic Lyapunov function under small perturbation, which provide a good description of persistence. We observe that these assumptions on the functional response are relative weak and valid for many forms of growth response.

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1. Introduction

The classic chemostat with single species and single substrate is described by

$$\begin{cases} \frac{dS(t)}{dt} = D(S^0 - S(t)) - \frac{1}{\delta}p(S(t))x(t), \\ \frac{dx(t)}{dt} = -Dx(t) + p(S(t))x(t), \end{cases} \quad (1.1)$$

where $S(t)$, $x(t)$ stand for the concentrations of nutrient and microbe at time t , respectively; S^0 is the original input concentration of nutrient and D represents the volumetric flow rate of the mixture of nutrient and microorganism, i.e. the dilution rate. The term $\frac{1}{\delta}p(S)$ denotes the uptake rate of substrate of the microbial population. $p(S)$ represents the per capita

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growth rate of the species and so δ is a growth yield constant. The growth response function $p : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is generally assumed that

$$p \text{ is continuously differentiable,} \tag{1.2}$$

$$p(0) = 0, \quad p(S) > 0 \text{ for } S > 0. \tag{1.3}$$

Butler et al. [1] investigated the global dynamics of the chemostat with a general class of functions describing nutrient uptake. As a simplified case (single-species), we can derive that there exists a uniquely defined positive real number $0 < \lambda \leq \infty$ such that

$$p(S) < D \text{ for } 0 < S < \lambda,$$

$$p(S) > D \text{ for } S > \lambda.$$

Here λ represents the break-even concentration of the substrate for the species $x(t)$. If $\lambda < S^0$, the solution of system (1.1) satisfies

$$\lim_{t \rightarrow \infty} S(t) = \lambda, \quad \lim_{t \rightarrow \infty} x(t) := x^* = \delta(S^0 - \lambda).$$

In other words, in the case of any monotone uptake functions (such as Monod functional response), when $p(S^0) > D$, then the critical point $E_\lambda = (\lambda, x^*)$ is globally asymptotically stable. The similar results are obtained in [2–4], which prove that the competitive exclusion principle holds in competition chemostat models with delays. There are also many extensive studies in the literature concerned with the dynamics of chemostat and we here do not mention them in detail (see [5–8] and the references therein).

Those important and useful works on deterministic chemostat provide a great insight into the effect of the dilution rate D , but in the real world, the natural growth of populations is inevitably affected by random fluctuations. Ecosystem dynamics are always subject to the environmental noise, thus it is more realistic to include the effect of stochasticity rather than to study models that are entirely deterministic. In fact, the random fluctuations are described by various types of environmental noise. Firstly, let us consider the white noise. The nutrient and microbial populations are usually estimated by an average value plus errors. We assume that the errors follow the normal distributions, while the standard deviations of the errors (known as the noise intensities) may depend on the population sizes. Therefore we add linear-perturbation terms $\sigma_1 S(t)dB_1(t), \sigma_2 x(t)dB_2(t)$ into the two equations respectively in the deterministic chemostat (1.1) (i.e. based on the approach used in [9] to include stochastic effects. Readers can also refer to [10], Appendix A to see the construction of this kind of stochastic model). Taking the white noise into account, microorganism systems described by the stochastic differential equations have recently been studied by many researchers [9,11–15]. When the perturbation is sufficiently large, the microbe population will be forced to expire whilst it remains stochastic permanent when the perturbation is small.

In view of the feasibility of varying the dilution rate D in the chemostat (see [16]), let us take a further step by considering another type of environmental noise, the colored noise, namely the telegraph noise. The telegraph noise can be illustrated as a switching between two or more environmental regimes which differ by factors such as humidity and temperature [17,18]. These environmental conditions are also important external factors for the continuous culture of microorganism in the chemostat. The introduction of regime switching into the dilution rate is realistic. The growth of microorganisms is evidently affected by external environment such as temperature, humidity and illumination. Once these factors change, we must adjust the dilution rate D to ensure the success in the continuous culture of microorganisms. The switching is memoryless and the waiting time for the next switch is exponentially distributed. Hence the regime switching is governed by a continuous-time Markov chain $r(t), t \geq 0$ with finite-state space $\mathbb{S} = \{1, 2, \dots, n\}$. Now the stochastic chemostat under regime switching becomes the following SDE model

$$\begin{cases} dS(t) = [D(r(t))(S^0 - S(t)) - \frac{1}{\delta}p(S(t))x(t)]dt + \sigma_1(r(t))S(t)dB_1(t), \\ dx(t) = [-D(r(t)) + p(S(t))]x(t)dt + \sigma_2(r(t))x(t)dB_2(t), \end{cases} \tag{1.4}$$

where $B_1(t), B_2(t)$ are independent standard one-dimensional Brownian motions with $B_i(0) = 0$ and $\sigma_i (\sigma_i^2 > 0), i = 1, 2$ are their intensities, respectively. For each $k \in \mathbb{S}$, parameter $D(k) > 0$. System (1.4) is operated as follows: if $r(0) = k_0$, the system parameters obey $D(r(t)) = D(k_0), \sigma_1(r(t)) = \sigma_1(k_0)$ and $\sigma_2(r(t)) = \sigma_2(k_0)$ until time τ_1 when the Markov chain jumps to k_1 ; the system parameters will then obey $D(k_1), \sigma_1(k_1)$ and $\sigma_2(k_1)$ from time τ_1 till time τ_2 when the Markov chain jumps to the next state k_2 . The system will continue to switch as long as the Markov chain jumps. Obviously τ_i is the stopping time. These stage systems at $t \in [\tau_i, \tau_{i+1})$ can be regarded as the subsystems of Eq.(1.4).

The population systems disturbed by both white and colored noise have been researched by many authors. Takeuchi et al. [19] studied a 2-dimensional autonomous predator–prey Lotka–Volterra system with regime and revealed that if two equilibrium states of the subsystems are different, all positive trajectories of this system always exit from any compact set of \mathbb{R}_+^2 with probability one; while if the two equilibrium states coincide, then the trajectory either leaves any compact set of \mathbb{R}_+^2 or converges to the equilibrium state. Luo and Mao [20] studied the stochastic population dynamics under regime

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