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EM algorithm for one-shot device testing with competing risks under exponential distribution

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ABSTRACT

This paper provides an extension of the work of Balakrishnan and Ling [1] by introducing a competing risks model into a one-shot device testing analysis under an accelerated life test setting. An Expectation Maximization (EM) algorithm is then developed for the estimation of the model parameters. An extensive Monte Carlo simulation study is carried out to assess the performance of the EM algorithm and then compare the obtained results with the initial estimates obtained by the Inequality Constrained Least Squares (ICLS) method of estimation. Finally, we apply the EM algorithm to a clinical data, ED01, to illustrate the method of inference developed here.

1. Introduction

One-shot devices often have multiple components that can cause failure. For example, a fire extinguisher contains a cylinder, a valve and chemicals inside; an automobile air bag contains a crash sensor, an inflator and an air bag; and for any packed food (which is also a kind of one-shot device), there are different causes for food expiry such as the growth of microorganism in the package, the moisture level and the food deterioration due to oxidation. A failure of any of the components will result in the failure of the product. However, by the very nature of one-shot devices, each product can be used only once at a specified time (Inspection Time, *IT*) and it gets destroyed afterwards. The outcome from each of the devices is therefore binary, either left-censored (failure) or right-censored (success). The data, which is a collection of these outcomes, will be both left- and right-censored, an extreme form of interval censoring. For those failed units, we will normally check for the cause responsible for the failure. Thus, the information collected from a life-test on one-shot devices in this case will include the status of the unit at inspection time as well as the cause of failure in case the unit has failed.

The one-shot device testing considered in this work will be conducted in an accelerated life test (ALT) setting since we will be often interested in the reliability assessment of highly reliable products. If the products are tested under normal conditions, the failure times of the products will be very large, thus resulting in a very large testing time. ALT will shorten the lifetimes of products by increasing the stress levels, and we can use several stress factors such as temperature and humidity for this purpose. After estimating the parameters under high stress conditions, we can extrapolate the life characteristics such as mean life time and failure rates from high stress conditions to normal operating conditions; see Meeter and Meeker [2] and Meeker et al. [3].

Expectation Maximization (EM) algorithm will be adopted in this paper. It is a powerful technique for obtaining the Maximum Likelihood Estimates (MLEs) in the case of complicated likelihood with missing data. The data obtained from one-shot device testing are both left- and right-censored as mentioned above and so the missing information is usually large. The maximum likelihood estimators will not be in a closed-form. For this reason, the EM algorithm will be a natural way to handle the estimation problem in this case. It enables an efficient determination of the maximum likelihood estimates. Considerable work has been done on estimating the model parameters by the use of EM algorithm: Ng et al. [4] developed EM algorithm for log-normal and Weibull distribution based on progressively censored data; Nandi and Dewan [5] developed EM algorithm for estimating the parameters of the bivariate Weibull distribution under random censoring scheme; and Pal [6] has discussed an EM algorithm for cure rate models. The EM algorithm has also been employed in estimating the masked causes under a competing risk model; see Park [7] and Craiu and Duchesne [8].

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For and cost effectiveness as well as convenience, the devices in ALT are often inspected at certain discrete time points. ALT in this form is referred to as intermittent inspection. The one-shot model is a special case of intermittent inspection when each device under test will be inspected only once and gets destroyed afterwards. Sohn [9] provided a model and optimal test plans for analyzing the life characteristics from a stockpile of one-shot products under the logistic lifetime distribution. Yates and Mosleh [10] proposed a Bayesian method for the reliability estimation in an aerospace system. Newby [11] assumed the one-shot device having repairable components and developed a monitoring and maintenance procedure. Fan et al. [12] used Bayesian approaches for analyzing highly reliable one-shot devices. They suggested three priors for the Bayesian estimation: Exponential, Normal and Beta, and their simulation results show that all three priors perform similarly when the data possess enough information. However, if the data possess zero-failure cases, normal prior has been recommended. Fan and Chang [13] also focused on the zero-failure reliability test of high quality one-shot devices with Bayesian analysis. Chuang [14] developed statistical inference for the mean-time-to-failure and reliability of one-shot devices under the Weibull life time distribution. Yang [15] studied the one-shot device under a Brownian degradation process using the Bayesian approach. Balakrishnan and Ling [1] developed an EM algorithm for the estimation of parameters of one-shot device model under exponential distribution with a single stress factor, and Balakrishnan and Ling [16] further extended it to multiple stress factors. Balakrishnan and Ling [17,18] developed an EM algorithm for the same problem under Weibull and gamma lifetime distributions, respectively.

Some work has been done on the analysis of system with binomial subsystems and components, but not under the ALT form. Martz et al. [19] analyzed series systems of binomial sub-systems and components in a Bayesian setup without the linkage to the stress levels. Martz and Wailer [20] considered a more complex system that includes both series and parallel components.

However, none of these works consider the competing risks setting. With competing risks, the model becomes more complicated than all those considered earlier and so the corresponding estimation problem becomes quite complex. But, this competing risks model is more realistic since many one-shot devices do contain multiple components that could cause the failure of the device. This is the motivation for us to consider here the one-shot device model with competing risks.

In this paper, we will first specify the one-shot device testing model with competing risks in Section 2. We will assume that the lifetime distribution is exponential and that there are no masked causes of failure in the data. For convenience, we will confine our attention to the case of two competing risks corresponding to the failure of each device and then develop the EM algorithm in Section 3. The extension to the case of multiple competing risks can be done in a natural way. For evaluating the performance of the developed EM algorithm, a simulation study is conducted in Section 4. In Section 5, the proposed method is compared with the Fisher-scoring method. The modification of the algorithm for handling data with masked cause of failure is then discussed in Section 6. An illustrative example with a set of clinical data, ED01, is analyzed in Section 8. Finally, some concluding remarks are made in Section 9.

2. Model specification

Let us consider the testing of electro-explosive devices. The structure of an ordinary electro-explosive device is displayed in Fig. 1. Let us now assume that there are only two causes responsible for the failure of detonation, say, burnout of resistance wires (part 20 in Fig. 1) as Cause 1 and leakage of organic fuel (part 6 in Fig. 1) as Cause 2. An accelerated life test for such one-shot devices is set up as follows:

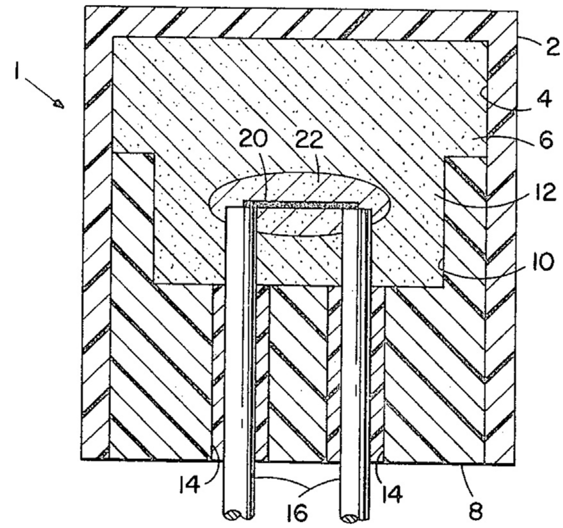

Fig. 1. An electro-explosive device designed by Thomas and Betts [21].

1. the tests are only checked at inspection times IT_i, for $i=1,\dots,I$,
2. the devices are tested under different temperatures (as stress levels) w_j, for $j=1,\dots,J$,
3. there are K_{ij} devices tested at IT_i and w_j,
4. the number of devices failed due to the rth cause at IT_i and w_j is denoted by D_{rij}, for $r=1,\dots,R$,
5. the number of devices that survive (successfully detonated) at IT_i and w_j is denoted by $S_{ij}=K_{ij}-\sum_{r=1}^{R} D_{rij}$.

Let us denote the random variable for the failure time due to Causes 1 and 2 by T_{rijk}, for $r=1,2$, $i=1,\dots,I$, $j=1,\dots,J$ and $k=1,\dots,K_{ij}$, respectively. In this work, we assume that T_{rijk} are independent of each other and follows exponential distribution with rate parameter λ_{rj} with p.d.f.

$$f_{rj}(t)=\lambda_{rj}e^{-\lambda_{rj}t}, r=1,2, j=1,\dots,J,$$

and with c.d.f.

$$F_r(IT_i|w_j)=\int_0^{IT_i} f_{rj}(t)\,dt=1-e^{-\lambda_{rj}IT_i}, r=1,2, i=1,\dots,I, j=1,\dots,J,$$

where λ_{rj} is the failure rate of the rth component in the device under temperature w_j. Of course, t_{rijk} will be used to denote the realization of the r.v. T_{rijk}. The relationship between λ_{rj} and w_j is assumed to be a log-link function of the form

$$\lambda_{rj}(\boldsymbol{\alpha})=\alpha_{r0}\exp(\alpha_{r1}w_j), \quad \alpha_{r0},\alpha_{r1},w_j\geq 0. \tag{1}$$

We define Δ_{ijk} to be the indicator for the kth device under temperature w_j and inspection time IT_i. When the device is successfully detonated, we will set $\Delta_{ijk}=0$. However, if the device fails to detonate, we will identify (by careful inspection) the specific cause responsible for the failure. If Risk r is the cause for the failure, we will denote this event by $\Delta_{ijk}=r$, for r=1,2. Mathematically, the indicator Δ_{ijk} is then defined as

$$\Delta_{ijk}=\begin{cases}0 & \text{if } \min(T_{1ijk},T_{2ijk})>IT_i,\\ 1 & \text{if } T_{1ijk}<\min(T_{2ijk},IT_i),\\ 2 & \text{if } T_{2ijk}<\min(T_{1ijk},IT_i),\end{cases} \tag{2}$$

and then δ_{ijk} will be used for the realization of Δ_{ijk}.

For example, if we conduct the ALT under temperatures $w_j=35,45,55,65\,^\circ$C and with inspection times $IT_i=10,20,30$