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Modelling and supervisory control of hybrid dynamical systems via fuzzy l-complete approximation approach

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HIGHLIGHTS

- A fuzzy l-complete approximation approach is proposed for abstraction of hybrid systems.
- The comparative results show that some disadvantages of the l-complete approximation are overcome.
- Supervisory control theory is applied to hybrid system based on its discrete abstraction.
- Regulation is realized using of discrete controller.

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ABSTRACT

This paper proposes a fuzzy l-complete approximation technique based on Willems' behavioural systems theory for a class of hybrid system with symbolic inputs and quantized measurements. The discrete controller (or supervisor) is designed based on the abstracted (or approximated) system and high-level specifications. Furthermore, we show that the set of external behaviour (i.e., state set) of the fuzzy l-complete approximation based on the notion of '*guardian states*' contains or at least equal to the behaviour set generated by traditional l-complete discrete abstraction. Finally, several simulation examples are presented to show the effectiveness of the proposed method for supervisory control of a special class of hybrid systems.

1. Introduction

In the past two decades, hybrid system has become an increasingly important subject in the field of control systems. In real-world large-scale systems, there are usually interactions between different components in different forms, such as continuous plant output and discrete control signals. How to reduce the complexity of those hybrid systems becomes a crucial research issue.

There are several modelling formalisms for hybrid system control problems. Some researchers modelled the problem based on continuous component and used the discrete part as a regulator, e.g. switched nonlinear system in [1] and fuzzy modelling for hybrid systems in [2–4]. Recently much work has been done in hybrid system abstraction (or approximation) and controller design, e.g., in [5] and [6]. However, for certain systems, the specifications can only be defined in terms of symbolic variables, or, like Sloth and Wisniewski pointed out in [7], "*no system trajectories can reach an unsafe subset of*

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the state set". Design of hybrid controllers from high-level specifications using abstraction-based approaches has gained popularity over the past two decades, such as [8] and [9].

Consider a class of hybrid control problems: the plant state evolves in $\mathbb{R}^n$ and is influenced by unknown yet bounded real-valued disturbances, whereas control inputs (sometimes measurement signals) are discrete-valued or symbolic (e.g. open/close valves for batching-tanks in chemical processes). We can convert such hybrid systems into a purely discrete one to directly apply DES approaches, e.g., in [10–12]. To deal with this class of systems, several methods were proposed, such as [13–15]. Among them, the *l*-complete approximation method proposed in [10] is distinct in: (1) the accuracy of abstraction can be improved by only tuning the parameter *l* without introducing any other external signals (this nice property was utilized in [16]) and (2) behavioural framework is used to model the controlled plant [17]. However, according to [18], there are still certain disadvantages: the state set in *l*-complete approximation is large and the behaviour set deduced by it is larger than that deduced by continuous system.

Supervisory control was originally proposed by Ramadge and Wonham in [19] and [20] for discrete event dynamic systems (DEDS). Since then it has been widely applied to both DEDS and hybrid systems [21–24]. In the perspective of fuzzy sets theory, the symbolic inputs and specifications can be more naturally and properly represented by linguistic variables [25–27]. In [28], the authors developed fuzzy model predictive control method for nonlinear hybrid systems with discrete inputs. Their results suggest that by suitably defining the cost function, satisfactory control can be attained. A strategy for the construction of a discrete finite state automaton based on fuzzy partition was proposed in [29]. Those previous work inspired us to apply fuzzy sets theory to improve the *l*-complete approximation. Based on this idea, a fuzzy *l*-complete approximation method arises, which can be considered as an extension of the original *l*-complete approximation method. By use of the concept of "Guardian state" in the area of hybrid automata, the meaningless states in an otherwise large state set can be removed.

The rest of the paper is organized as follows. The preliminaries were given in Section 2. In Section 3, *l*-complete approximation method is briefly reviewed. Fuzzy *l*-complete approximation method is described in Section 4. The supervisory control theory is briefly introduced in Section 5. Section 6 presents several illustrative examples. Finally, Section 7 concludes the whole paper.

2. Preliminaries

In this Section, the mathematical notations and preliminaries are introduced. All I/O signals of the continuous or hybrid systems under study are discrete-time in default, i.e., $\{t_0, t_1, \ldots\}$ with $\Delta T = t_{k+1} - t_k$ being a constant sampling interval. Thus, hereafter the terms "continuous" and "discrete" will be only used to indicate the codomain of signals. For example, the codomain of a *continuous* signal may be the set of real values $\mathbb{R}^n$, whereas that of a discrete signal may be a finite set of symbols, such as {"open", "closed"}, {"low", "medium", "high"}. The continuous and discrete variables are indicated by the subscript "c" and "d", respectively. To simplify notation, k and $k+1$ were used to indicate t_k and t_{k+1}, respectively, without causing confusions.

2.1. Model of continuous plant

As in [13], the continuous plant can be modelled as the following discrete-time dynamic system:

$$\mathbf{x}(k+1) = f(\mathbf{x}(k), \mathbf{n}(k), u_d(k)) \tag{1}$$

$$y_d(k) = q_y(\mathbf{x}(k)) \tag{2}$$

where $k \in \{0, 1, 2, \ldots\}$ is the time index, $x(k) \in \mathbb{R}^n$ the state at time k, $\mathbf{n}(k) \in \mathbb{R}^r$ an unknown but bounded noise: $\mathbf{n}(k) \in \mathbf{N} := \{\mathbf{n} \,|\, \mathbf{n} \in \mathbb{R}^r, \|\mathbf{n}\|_\infty \le 1\}$ with $\|\mathbf{n}\|_\infty = \max_i |n_i|$, $u_d(k) \in U_d$ and $y_d(k) \in Y_d$ are control input and measurement, respectively, the sets $U_d = \left\{u_d^{(1)}, \ldots, u_d^{(\alpha)}\right\}$ and $Y_d = \left\{y_d^{(1)}, \ldots, y_d^{(\gamma)}\right\}$ are finite with $\alpha, \gamma < \infty$, $f : \mathbb{R}^n \times \mathbb{R}^r \times U_d \to \mathbb{R}^n$ is the state transition map, and $q_y : \mathbb{R}^n \to Y_d$ the output map.

Eq. (2) can be also rewritten into

$$y_c(k) = q_y(\mathbf{x}(k)) \tag{3}$$

$$y_d(k) = q_d(y_c(k)) \tag{4}$$

in order to represent systems with discrete (finite) external input signals and continuous output in the same form. In the sequel, we use symbol y_d to denote the measurement consistently.

If the plant is hybrid, i.e., it consists of a continuous part and a discrete part, we could model the continuous part with discrete abstraction and then combine the discrete abstraction with the discrete part [18]. The general configuration of the hybrid control system is depicted in Fig. 1, where the dashed box represents the controlled plant (a hybrid dynamic system with symbolic inputs and quantized measurements) and u_d is the symbolic input (external events) to the component of DEDS in hybrid system. The evolution of continuous-variable dynamic system (CVDS) might also generate "internal events" to trigger the DEDS. The continuous output of CVDS, y_c, and the output of the DEDS are input to an Event Generator to obtain the discrete (symbolic) output y_d.