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# $\mathcal{H}_\infty$ performance for neutral-type Markovian switching systems with general uncertain transition rates via sliding mode control method

Jing Xie [a], Yonggui Kao [b,c], Ju H. Park [c,*]

[a] College of Automation Engineering, Qingdao University of Technology, Qingdao, 266520, PR China
[b] Department of Mathematics, Harbin Institute of Technology (Weihai), Weihai, 264209, PR China
[c] Department of Electrical Engineering, Yeungnam University, 280 Daehak-Ro, Kyongsan 38541, Republic of Korea

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ABSTRACT

The paper is devoted to the investigation of observer-based $\mathcal{H}_\infty$ sliding mode controller design for a class of nonlinear uncertain neutral Markovian switching systems (NUNMSSs) with general uncertain transition rates. In this case, each transition rate can be completely unknown or only its estimate value is known. Firstly, an $\mathcal{H}_\infty$ non-fragile observer subjected to the general uncertain transition rates of the NUNMSSs is constructed. By some specified matrices, the connections among the designed sliding surfaces corresponding to every mode are founded. Secondly, the state-estimation-based sliding mode control law is designed to guarantee the reachability of the sliding surface in a finite time interval. Thirdly, an asymptotically stochastic stability criterion is established by utilizing a new-type Lyapunov function to guarantee the error system and sliding mode dynamics to be asymptotically stochastic stable with a given disturbance attenuation level. Finally, an example is provided to illustrate the efficiency of the proposed method.


## 1. Introduction

Recently, Markovian jump systems (MJSs), have received increasing interests, see [1–31] and the references therein, because many physical systems may cause abrupt variations in their structure, due to random failures or repairs of components, sudden environmental disturbances, changing subsystem interconnections, and abrupt variations in the operating points of a nonlinear plant. Kao et al. studied non-fragile observer based $\mathcal{H}_\infty$ sliding mode control for Itô stochastic systems with Markovian switching and Markovian neutral-type stochastic systems in [20] and [21], respectively. However, the transition rates (TRs) in the above mentioned literature are assumed to be completely known. Generally speaking, it is hard to precisely estimate all the TRs in some jumping processes because of expensive cost or other factors. Therefore, analysis and synthesis problem for MJSs with uncertain TRs, including bounded TRs and partly unknown TRs, have attracted increasing interests [22–31]. Karan et al. [22] inspected the stochastic stability robustness for continuous-time and discrete-time Markovian jump linear systems (MJLSs) with upper bounded TRs. Zhang and Boukas [23] explored stability and stabilization for the continuous-time MJLSs with partly unknown TRs. Zhang et al. [24] derived necessary and sufficient conditions for analysis and synthesis of MJLSs with incomplete transition descriptions. Zhang and Boukas [25] looked into mode-dependent $\mathcal{H}_\infty$ filtering for discrete-time MJLSs with partly unknown transition probability. Partly unknown TRs for MJLSs were also involved in [26–31]. Wei et al. [30] developed a new $\mathcal{H}_\infty$ filtering design for continuous-time MJLSs with

* Corresponding author.
*E-mail address:* jessie@ynu.ac.kr (J.H. Park).

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time-varying delay and partially accessible mode information. Guo and Wang [31] proposed another description for the uncertain TRs, which is called general uncertain TRs. Some of the general uncertain TRs may be completely unknown or partially known, which makes the general uncertain TR model be applicable to more practical cases. Both bounded uncertain TR models and partly uncertain TR models are the special cases of general uncertain TR models.

To the authors' best knowledge, the problem of observer-based $\mathcal{H}_\infty$ sliding mode control for nonlinear uncertain neutral Markovian switching systems (NUNMSSs) with general uncertain TRs has not been well reconnoitered yet, which is still an open and challenging problem in many practical applications. Although the problem of stability on Markovian switching systems with nonlinearities has been worked out by now, the results about the sliding mode control mostly concentrated on the uncertain systems with linear input only. Therefore, the sliding mode control is utilized to Markovian switching systems with nonlinearity is still an open problem. In addition, we should notice that under Markovian switching, each mode of the system is dependent on each other via Markov process. Therefore, the following questions during the design of the sliding mode controller for NUNMSSs should be responded to:

Q1: If all the TRs are partially known, can we design the observers? What is the connection among the observers under Markovian switching for NUNMSSs with general uncertain TRs?

Q2: If all the TRs are partially known, can we design the sliding mode controllers? What is the connection among sliding functions under Markovian switching for NUNMSSs with general uncertain TRs?

Q3: How is the sliding mode controller updated to ensure the attraction of the sliding surface when the sliding surface changes from one to another under Markovian switching?

Motivated by the aforementioned discussions, we focus on the design of sliding mode control for a class of NUNMSSs with general uncertain TRs and will answer the above three questions in the following design (see Remarks 3, 4 and 5). In Section 2, the NUNMSS model with general uncertain TRs is formulated and some definitions and lemmas are stated. The parameter uncertainties, nonlinearities, and external disturbance are included in the system under consideration. Section 3 gives main results. Based on a non-fragile $\mathcal{H}_\infty$ observer, a robust sliding mode control law is established to guarantee the reachability of the sliding surface in finite time. In the design of sliding surface, a set of specified matrices are employed to establish the connections among sliding surfaces corresponding to each mode. It is shown that the state trajectories of the resultant NUNMSS with general uncertain TRs can be driven onto the specified sliding surfaces for each mode in finite time. The sufficient condition for the asymptotic stability of the overall closed-loop NUNMSSs with general uncertain TRs and a $\gamma$-disturbance attenuation level are derived in forms of LMIs by constructing a new-type Lyapunov function. In Section 4, an example is provided to illustrate the validity of the proposed method. Section 5 is the conclusion.

*Notations*: $\mathbf{R}^n$ and $\mathbf{R}^{n\times m}$ denote, respectively, the $n$-dimensional Euclidean space and the set of $n \times m$-dimensional real matrices. $\mathbf{N}^+$ denotes the sets of positive integers. $(\Omega, \mathbb{F}, \mathbb{P})$ is a complete probability space with a filtration $\{\mathbb{F}_t\}_{t\geq 0}$ satisfying the usual conditions. $\mathbf{L}^{\mathbb{P}}_{\mathbb{F}_0}([-\tau, 0]; \mathbf{R}^n)$ is the family of all $\mathbb{F}_0$-measurable $\mathbf{C}([-\tau, 0]; \mathbf{R}^n)$-valued random variables $\xi = \{\xi(\theta) : -\tau \leq \theta \leq 0\}$ such that $\sup_{-\tau\leq\theta\leq 0}\mathbb{E}\{\|\xi(\theta)\|_2^2\} < \infty$, where $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathbb{P}$. The superscript T denotes the transpose, and the notation $X \geq Y$ (respectively, $X > Y$) where $X$ and $Y$ are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). $\mathbf{L}^2$ stands for the space of square integral vector functions. $\|\cdot\|$ refers to the Euclidean vector norm, and $*$ represents the symmetric form of matrix.

## 2. Preliminaries and problem formulation

Consider a class of uncertain stochastic neutral time-delay systems subjected to input nonlinearity and Markovian switching as follows

$$\begin{cases} \mathrm{d}[x(t) - Ex(t-\tau)] = [(A(\gamma_t) + \Delta A(\gamma_t))x(t) + (A_1(\gamma_t) + \Delta A_1(\gamma_t))x(t-\tau) \\ \quad + B\phi(u, \gamma_t) + f(x(t), \gamma_t) + G(\gamma_t)v(t)]\mathrm{d}t + g(x(t), t, \gamma_t)\mathrm{d}\omega(t), \\ y(t) = C(\gamma_t)x(t) + D(\gamma_t)x(t-\tau), \\ x(\theta, \gamma_0) = \varphi(\theta, \gamma_0), \theta \in [-\tau, 0], \gamma_0 \in \mathbb{S} \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $\varphi(\theta, \gamma_0) \in \mathbf{L}^{\mathbb{P}}_{\mathbb{F}_0}([-\tau, 0]; \mathbf{R}^n)$ is a compatible vector valued continuous function, $\phi(u, \gamma_t) \in \mathbf{R}^m$ is the control input, $f(x(t), \gamma_t) \in \mathbf{R}^n$ is the nonlinear disturbance input, $v(t) \in \mathbf{R}$ is the exogenous noise, $g(x(t), t, \gamma_t) \in \mathbf{R}^{n\times l}$ is the stochastic perturbations, $\omega(t) = [\omega_1(t), \ldots, \omega_l(t)]^{\mathrm{T}} \in \mathbf{R}^l$ is a Brownian motion defined on a complete space, $y(t) \in \mathbf{R}^q$ is the controlled output. $E$, $A(\gamma_t)$, $A_1(\gamma_t)$, $B$, $G(\gamma_t)$, $C(\gamma_t)$, and $D(\gamma_t)$ are all real constant matrices with appropriate dimensions. $\Delta A(\gamma_t)$ and $\Delta A_1(\gamma_t)$ are parameter uncertainties.

Let $\{\gamma_t, t \geq 0\}$ be a right-continuous Markov process on the probability space which takes values in the finite space $\mathbb{S} = \{1, 2, \ldots, N\}$ with the generator $\Lambda = (\pi_{ij})_{N\times N}$ (also called the jumping transfer matrix) given by

$$\mathbb{P}\mathrm{r}\{\gamma_{t+\Delta} = j|\gamma_t = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & j \neq i, \\ 1 + \pi_{ij}\Delta + o(\Delta), & j = i. \end{cases} \tag{2}$$

Here $\Delta > 0$ and $\lim_{\Delta\to 0}\frac{o(\Delta)}{\Delta} = 0$. $\pi_{ij} \geq 0$ is the TR from $i$ to $j$ if $i \neq j$, and $\pi_{ii} = -\sum_{j\neq i}\pi_{ij}$. As usual, we assume that the Brownian motion $\{\omega(t), t > 0\}$ is independent from the Markov process $\{\gamma_t, t \geq 0\}$.