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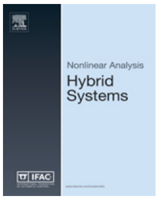

An optimal control approach to the design of periodic orbits for mechanical systems with impacts[☆]

Sara Spedicato [*], Giuseppe Notarstefano

Department of Engineering, Università del Salento, Via per Monteroni, 73100 Lecce, Italy

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ABSTRACT

In this paper we study the problem of designing periodic orbits for a special class of hybrid systems, namely mechanical systems with underactuated continuous dynamics and impulse events. We approach the problem by means of optimal control. Specifically, we design an optimal control based strategy that combines trajectory optimization, dynamics embedding, optimal control relaxation and root finding techniques. The proposed strategy allows us to design, in a numerically stable manner, trajectories that optimize a desired cost and satisfy boundary state constraints consistent with a periodic orbit. To show the effectiveness of the proposed strategy, we perform numerical computations on a compass biped model with torso.

1. Introduction

Hybrid systems, involving both continuous and discrete dynamics, arise naturally in a number of engineering applications. In particular, many robotics tasks, such as legged locomotion, (multi-finger) manipulation and load transportation with aerial vehicles, can be modeled as mechanical systems with impacts, a particular class of hybrid systems. These classes of systems experience continuous dynamics until an interaction with the surrounding environment (i.e., an impact) occurs, thus causing a state discontinuity due to impulsive forces and (possibly) a switch between different dynamics.

In this paper we concentrate on a particular aspect that has important implications both for modeling and control of robots, namely trajectory design.

Motivations. The trajectory design task has gained a lot of attention both as a preliminary task for control and as a tool to explore and understand the capabilities of the system [1,2] (see [3] for an earlier reference). Optimization techniques allow us to design reference trajectories for robot controllers by minimizing a given cost function. Typical cost functions are (i) the distance from a desired state-input curve (which does not satisfy the dynamics) and (ii) the energy injected into the system. For example, for humanoid robot design, the distance from a desired (but unfeasible) human-like walking pattern [4,5] is often considered. Additional challenges arise when the trajectory generation problem is addressed for (underactuated) mechanical systems with impacts. The impact events complicate the trajectory optimization problem since discontinuous changes in the state occur. For some systems, the underactuation and the instability of the continuous dynamics render the problem even more challenging. The optimal control theory offers powerful tools to deal with trajectory generation problems for hybrid dynamical systems.

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[*] Corresponding author.
E-mail addresses: sara.spedicato@unisalento.it (S. Spedicato), giuseppe.notarstefano@unisalento.it (G. Notarstefano).

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Literature review. The literature on optimal control of hybrid systems is quite vast, thus we report only two sets of contributions relevant for our work.

First, a general overview on recent advances for optimal control of hybrid systems is presented. A recent survey, focusing in particular on switched systems, is [6]. In [7] the authors propose optimal control algorithms for discrete-time linear hybrid systems which "combine a dynamic programming exploration strategy with multiparametric linear programming and basic polyhedral manipulation". In [8] a set of necessary conditions is formulated and optimization algorithms are presented for optimal control of hybrid systems with continuous nonlinear dynamics and with autonomous and controlled switchings. The algorithm described in [9] (for hybrid systems with partitioned state space and autonomous switching) is based on a version of the minimum principle for hybrid systems providing optimality conditions for intersections and corners of switching manifolds, and thus avoiding the combinatorial complexity of other algorithms (e.g., [8]). Furthermore, the work in [10] deals with optimal mode-scheduling via a gradient descent algorithm for the particular class of autonomous switched-mode hybrid dynamical systems. In [11,12] the design of switching laws for switched systems with linear dynamics, based on the optimization of a quadratic criterion, is addressed.

Second, we focus on trajectory optimization for the particular class of mechanical systems with impacts. Besides results regarding systems with impulsive controls, e.g. [13,14], we focus on the trajectory design for systems controlled by inputs of the continuous dynamics. The majority of works adopt parametric optimization methods, as, e.g., [15–17]. This means that trajectories are approximated as, for example, classical, trigonometric or Bézier polynomials and the optimization is performed with polynomial coefficients as decision variables. In other works, e.g. [18], the optimization problem is addressed via dynamics equation discretization and the optimal periodic trajectory is computed by means of an approximated cost function. Available software toolboxes are used to solve trajectory generation problems addressed in [15–18]. Recently, the optimization over jump times and/or mode sequence, as e.g., in [19,20], has been considered. In [19] the instants of jump are optimized, but the sequence of continuous dynamics modes is fixed. In [20] the mode sequence is also optimized by sequential quadratic programming. Focusing in particular on the trajectory generation problem for legged robots (the major robotic application regarding mechanical systems with impacts), challenges arising when dealing with underactuated robots are addressed in [21,22] by considering an additional "virtual" input acting on the unactuated degrees of freedom. Then, an optimal approximated trajectory of the underactuated dynamics is computed by dynamics inversion. Furthermore, online trajectory optimization is addressed in [23] through a method based on iterative LQG and in [24] trajectories are designed by using an auxiliary system of differential equations and then stabilizing the generated curves through a feedback on the target system.

Contributions. As main contribution of this paper, we develop an optimal control based strategy to design periodic orbits for a class of hybrid dynamical systems with impacts. First, we formulate an optimal control problem with ad-hoc boundary conditions. These ones are provided by studying how the initial conditions and the jump conditions are related in a periodic orbit with one jump per period. Second, instead of using available software, we develop an ad-hoc strategy based on the combination of trajectory functional optimization with three main tools: dynamics embedding, constraint relaxation and zero finding techniques. These tools enable us to (i) deal with the underactuated nature of dynamics, (ii) consider highly non trivial constraints and (iii) avoid the tedious search for a suitable "initial (guess) trajectory" to initialize the optimization algorithm. Furthermore, optimal control problems involved in our strategy are solved by combining the penalty function approach [25] with the Projection Operator Newton method [26]. In contrast with many strategies reported in the literature, we do not resort to approximations such as considering discrete sets of motion primitives (thus ending up with the only optimization of their parameters), and/or discrete time. In fact we consider system states and inputs as optimization variables and a second-order approximation of the optimization problem is directly constructed in continuous time. In detail, the proposed strategy is based on the following steps. By adding a fictitious input, we embed the system into a completely controllable (fully actuated) one. On this system we set up, by constraint relaxation, an unconstrained optimal control problem to find a trajectory of the system (a curve satisfying the dynamics) that minimizes a weighted L_2 distance from a desired curve. The desired curve, together with the weights of the cost functional, become important parameters in the designer's hand to explore the system dynamics, that is different periodic orbits that the system can execute. We solve the optimal control problem to generate a trajectory of the fully actuated system with almost null fictitious input. Differently from [21,22], we use this trajectory to initialize the last step of our strategy, where the optimal control problem is set up on the underactuated dynamics. A Newton update rule on the final penalty of the weighted cost functional is adopted in order to hit the desired final state. It relies on the solution of an optimal state transfer problem presented in [27], where the advantages of the method are highlighted with respect to the classic ones.

We provide a set of numerical computations showing the effectiveness of the proposed strategy. In particular, we generate a periodic gait for a three-link biped robot (compass model with torso). We perform two computations by choosing two different sets of weights. In the first scenario we try to generate a trajectory that is as close as possible to the guessed state curve. Vice-versa, in the second one, we compute a trajectory that minimizes the input effort (i.e., some sort of minimum-energy trajectory). It is worth noting that, although the three links model is relatively simple, its underactuation represents a significant challenge. Also, it is well known that for many applications, even such a reduced model of the biped dynamics is instrumental for analyzing and controlling the actual system, see, e.g., [24].

Paper organization. The paper is organized as follows. In Section 2 we introduce the model of the particular class of hybrid system we study in this paper and we present the problem formulation for the generation of periodic orbits. In Section 3 we