Nonlinear Analysis: Hybrid Systems 23 (2017) 122–141

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Dynamics analysis of a pest management prey–predator model by means of interval state monitoring and control

Yuan Tian [a,*], Tonghua Zhang [b], Kaibiao Sun [c]

[a] School of Information Engineering, Dalian University, Dalian 116622, China
[b] Department of Mathematics, Swinburne University of Technology, Hawthorn VIC 3122, Australia
[c] School of Control Science and Engineering, Dalian University of Technology, Dalian 116024, China

ARTICLE INFO

Article history:
Received 5 January 2016
Accepted 19 September 2016

Keywords:
Interval impulsive control
Integrated pest management
Parameter optimization
Stability

ABSTRACT

In this work, a new pest management strategy by means of interval state monitoring is introduced into a prey–predator model, i.e. when the pest density exceeds the slightly harmful level but is below the damage level, the biological control is adopted in case of the predator density below a maintainable level, once the pest density exceeds the damage level, the chemical control is adopted. In order to determine the frequency of the chemical control and yield of releases of the predator, analysis on the existence of order-1 or order-2 periodic orbit is carried out by the construction of Poincaré map. The results could make the pest control strategy to be a periodic one without real-time monitoring the species. In addition, the stability and attractiveness of the periodic orbit are obtained by geometry approach, which ensures a certain robustness of control, i.e., even though the species densities are detected inaccurately or with a deviation, the system will be eventually stable at the periodic orbit under the control action. Furthermore, to obtain the optimum chemical control strength and yield releases of the predator, an optimization problem is constructed. The analytical results presented in the work are validated by numerical simulations for a specific model.

1. Introduction

Pest management is an interesting and significant issue in real life. The traditional and efficient method is to spray pesticide. However, unrestrained use of persistent pesticide not only increases the incidence of pesticide-resistant pest varieties, but also inflicts harmful effects on humans through the accumulation of hazardous chemicals in their food chain [1]. Moreover, pesticide pollution is a major threat to beneficial insects, which sometimes are more sensitive to pesticides than target pest. So if it is not necessary, the chemical control should not be easily considered.

Biological control, as an alternative control method, by releasing biological agents to increase their effectiveness plays an important role in suppressing pests growth [2,3]. Researches on augmentation as a biological control method have also shown that some practices are cost-effective [4] and others are not and sometimes can have disastrous consequences without being well planned [5]. Especially, when encountering the situation of some disaster, which is limited in a small range, the method is not very effective or ideal at this time.

Integrated pest management (IPM) is an effective method in controlling pests with minimal use of harmful pesticides and other undesirable measures, which has been proved to be more effective than the classic methods both experimentally [1,2]

* Corresponding author.
E-mail address: tianyuan1981@163.com (Y. Tian).

http://dx.doi.org/10.1016/j.nahs.2016.09.002

and theoretically [6,7]. Many researchers have introduced IPM strategy in modeling pest control, for example: the periodic release of predators and infected pests [8,9]; the periodic release of infected pests combined with periodic applications of pesticides [10]; the periodic release of predators, pests combined with periodic applications of pesticides [11–16] and state dependent release of predators combined with applications of pesticides [17–23].

By considering biological control and chemical control adopted at different pest levels, Nie et al. [24,25] studied two prey–predator models with twice impulsive controls. The idea is interesting, but the suppressing effect of natural predator is neglected at the biological control level. Based on this consideration, Tian et al. [26,27] introduced a predator density level into the models in [24,25], i.e. when the predator density in the system is below the level, the biological control strategy is adopted until the predator density is higher than that level. And followed, by choosing different predator control level, Zhao et al. [28] and Zhang et al. [29] proposed and analyzed the dynamics of other type of predator–prey models in detail.

The idea of involving biological and chemical controls at different prey densities is interesting and has certain practical significance. Before the chemical control has to be adopted, a biological control should be adopted in advance, which can extend the time for pest density to reach the pest damage level. However, there exists a key problem in this process that needs to be dealt with properly, i.e. the biological control is adopted when the pest density reaches the first control level and the predator density is lower than its maintainable level, but for a higher pest density, no control strategy is adopted. This is obviously unreasonable. Since the biological control and chemical control are taken at different pest levels, a more reasonable model should also consider the control action when the pest density lies between the two levels. Motivated by this control strategy, Tian [30] proposed a pest management Gompertz model with interval state feedback impulsive control. As a continuation, a prey–predator Logistic model by means of interval state monitoring and control is presented and control optimization is carried out based on the qualitative analysis on the proposed model.

This paper is organized as follows. In Section 2, a pest control prey–predator model by means of interval state monitoring and control is put forward, and some basic definitions are given. In Section 3, a detail dynamics analysis in case of the chemical control strength is carried out. In Section 4, numerical simulations are presented with a specific model to verify the theoretical results step by step. Finally, conclusions are presented in Section 5.

2. Model formulation and preliminaries

2.1. Model formulation

Let $x(t)$ and $y(t)$ denote the pest and its natural enemy densities at time t, which follow the logistic model

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = x(t)\left[r - \dfrac{rx(t)}{K} - by(t)\right] \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = y(t)[\varrho bx(t) - d] \end{cases} \tag{2.1}$$

where $r > 0$ is the birth rate, $K > 0$ is the environmental carrying capacity for the prey in absence of predator, $b > 0$ is the contact rate, $0 < \varrho < 1$ is the conversion coefficient and $0 < d < 1$ is the death rate of predator. In this study, it is assumed that

$$(P1) : K > \overline{K} \triangleq \frac{d}{\varrho b}.$$

Let ET_1 denote the first pest control level, ET_2 $(ET_1 < ET_2 < K)$ denote the pest damage level above which the chemical control is taken to suppress pests increasing. When the prey density locates in between ET_1 and ET_2 (i.e. $x(t) \in [ET_1, ET_2)$) and the predator density is below a level $\overline{y}^{\lambda}_x$, only the biological control is taken as a control method to suppress the prey. By the biological background, it is only necessary to consider the dynamic behavior of such a system in the region $\Omega = \{(x, y)|0 < x \le ET_2, 0 < y \le y_M - \varrho x\}$, where y_M is a sufficiently large constant satisfying $\mathrm{d}\chi_0/\mathrm{d}t|_{\chi_0=0} < 0$ where $\chi_0 : \varrho x + y - y_M = 0$. According to the above mentioned control strategy, the system can be modeled by the following impulsive differential equations:

$$\begin{cases} \left.\begin{aligned} &\dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = x(t)\left[r - \dfrac{rx(t)}{K} - by(t)\right] \\ &\dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = y(t)[\varrho bx(t) - d] \end{aligned}\right\} & \begin{aligned} &x < ET_1 \\ &\text{or } x \in [ET_1, ET_2), y > \overline{y}^{\lambda}_x \end{aligned} \\ \left.\begin{aligned} &\Delta x = 0 \\ &\Delta y = \alpha(x(t)) \end{aligned}\right\} & x \in [ET_1, ET_2), 0 < y \le \overline{y}^{\lambda}_x \\ \left.\begin{aligned} &\Delta x = -px(t) \\ &\Delta y = -qy(t) \end{aligned}\right\} & x = ET_2, 0 < y \le \overline{y}^{\lambda}_x \end{cases} \tag{2.2}$$

where $\overline{y}^{\lambda}_x$ is the predator maintainable level (or critical biological control level) at the pest level x, which is defined as follows

$$\overline{y}^{\lambda}_x = \overline{y}^{\lambda}(x) \triangleq (r(K - \overline{K})/bK)\lambda(x), \tag{2.3}$$