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On stability of discrete-time switched systems

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ABSTRACT

This article deals with stability of discrete-time switched systems. Given a family of nonlinear systems and the admissible switches among the systems in the family, we first propose a class of switching signals under which the resulting switched system is globally asymptotically stable. We allow unstable systems in the family and our stability condition depends solely on asymptotic behaviour of the switching signals. We then discuss algorithmic construction of the above class of switching signals, and show that in the presence of exogenous inputs and outputs, a switching signal so constructed also ensures input/output-to-state stability for discrete-time switched nonlinear systems. We finally show that our class of switching signals that ensures global asymptotic stability also extends to the continuous-time setting with minor modifications under standard assumptions. We employ multiple Lyapunov-like functions and graph theoretic tools as the main apparatuses for our analysis.

1. Introduction

A *switched system* [1, Chapter 1] has two components—a family of systems and a switching signal. The systems in the family are described by a collection of indexed differential or difference equations. The *switching signal* selects an *active subsystem* at every instant of time, i.e., the system from the family that is currently being followed. Switched systems are employed to model real-world dynamical systems that are subject to known or unknown abrupt parameter variations [2, p. 5] such as networks with periodically varying switches, and/or sudden changes of system structures due to faults or other reasons, such as the failure of a component or a subsystem that may have occurred in so short a time interval as to be considered an instantaneous event in comparison to the nominal time constants of the plant model. As a result, such systems find wide applications in automotive control, network and congestion control, etc. [2, p. 5].

Switching signals are broadly classified as time-dependent (depends only on time), state-dependent (depends on the states as well), and with memory (also dependent on the history of the active subsystems) [3, Section I]. Qualitative behaviour of switched systems depends not only on the behaviour of the individual subsystems in the family, but also on the switching signal. For instance, divergent trajectories can be generated by switching appropriately among stable subsystems, while a suitably constrained switching signal may ensure stability of a switched system even if all the subsystems are unstable. Due to such interesting features, stability of switched systems has attracted considerable research attention over the past few decades, see e.g., [4–7,3] for detailed surveys on the existing literature.

Stability of switched systems is broadly classified into two categories—stability under *arbitrary switching* [1, Chapter 2] and stability under *constrained switching* [1, Chapter 3]. In case of the former, conditions on the family of systems are

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identified such that the switched systems generated under all admissible switching signals are stable, while in case of the latter, given a family of systems, conditions on the switching signals are identified such that the corresponding switched systems are stable. In this article our focus is on stability of discrete-time switched systems under constrained switching. Constraints on the switching signals may arise from physical constraints on the systems, e.g., in automobile gear switching, a particular sequence is to be followed, or a priori knowledge about the possible switching logic in the system may be present, e.g., partitions of the state space and their induced switching rules, etc., see [3, Section III] for an extended discussion.

We consider a family of discrete-time nonlinear systems to be given and our focus is on identifying classes of stabilizing time-dependent switching signals under which the resulting switched system is stable. To wit, we are interested in switching signals for which a "decision" to switch depends only on time, and does not consider values of the system states. Prior results on stability of switched systems under constrained switching has primarily utilized the concept of *slow switching* vis-a-vis (*average*) *dwell time switching*. Intuition suggests that a switched system whose constituent subsystems are all stable, would itself be stable provided that the switching is slow. Indeed, the basic idea of stability under slow switching is that if all the subsystems are stable and the switching is sufficiently slow, then the "energy injected due to switching" gets sufficient time for dissipation due to the stability of the individual subsystems. This idea is captured to some extent by the concepts of dwell time and average dwell time switching [1, Chapter 3], [3]. In case of dwell time switching, a minimum duration of time is maintained between any two consecutive switches [1, Section 3.2.1]. A more general class of switching signals, namely, those with an average dwell time [1, Section 3.2.2] allows the number of switches on any interval of time to grow at most as an affine function of the length of the interval, and stability analysis of switched systems under these switching signals are standard in the literature. The underlying idea is that stability of the switched system is preserved even if the switching signal admits fast switching, provided that the switches do not accumulate too quickly. Although the concepts of (average) dwell time switching were originally developed in the continuous-time setting [8,9], these extend readily to the discrete-time setting with the (average) dwell time expressed in terms of the number of time steps [3]. However, in the presence of unstable systems in the family, slow switching alone is not sufficient to guarantee stability, and additional conditions are required to ensure that the switched system does not spend too much time in the unstable components [3].

Exponential stability of discrete-time switched linear systems in the presence of unstable subsystems was studied in [10] (which is a discrete-time analog of [11]). A class of stabilizing switching signals was characterized involving a modified definition of average dwell time and a method of activating the Schur stable subsystems (if any) arbitrarily but activating the unstable subsystems depending on a pre-specified ratio. In [12] we identified a class of switching signals for global asymptotic stability (GAS) of discrete-time switched linear systems. The stabilizing condition involves only asymptotic behaviour of the switching signal and do not rely on point-wise bounds on the number of switches and duration of activation of the unstable subsystems unlike in the case of average dwell time switching. However, the presence of at least one asymptotically stable system in the family is required. Input-to-state stability (ISS) of discrete-time switched systems under average dwell time switching was studied in [13] (which is a discrete-time analog of [14]). It was shown that if all the subsystems are ISS and their ISS-Lyapunov functions satisfy standard assumptions, then the switched system has ISS under switching signals obeying a sufficiently large average dwell time. Recently, in [15] (which is a discrete-time analog of [16]), we discussed a discrete-time analog of an ISS version of [17, Theorem 2]. Given a family of systems such that not all subsystems are ISS, a class of stabilizing switching signals is characterized based on average dwell time switching and constrained point-wise activation of the unstable subsystems.

In this article our contributions are the following:

- Our first contribution is towards identifying a class of switching signals under which a discrete-time switched system is globally asymptotically stable. Given a family of discrete-time nonlinear systems (without inputs), we propose a class of stabilizing switching signals under which the resulting switched system is GAS. This result is an extension of [12, Theorem 1] to the case of discrete-time switched nonlinear systems under standard assumptions.
- We then consider the algorithmic construction of the above class of stabilizing switching signals from [12,18]; and show that given a family of discrete-time nonlinear systems (with inputs and outputs), a switching signal constructed as above ensures input/output-to-state stability (IOSS) of the resulting switched system. Earlier in [19] we showed that such construction of a switching signal also guarantees ISS of a discrete-time switched system. This leads to a new class of switching signals ensuring IOSS of discrete-time switched systems that affords simple algorithmic construction.
- Our third contribution is towards showing that the class of switching signals proposed for GAS of discrete-time switched systems extends to the case of continuous-time switched systems with minor modifications under standard assumptions.

The main features of the results presented in this article are the following:

- **GAS**:
  - Our stability conditions involve *only* certain asymptotic properties of the switching signals and do not rely on *point-wise* bounds unlike in the case of average dwell time switching signals. Consequently, there is a plenty of flexibility insofar as the transient behaviour of the switching signals is concerned.
  - We allow unstable systems in the family. On the one hand, our conditions require the presence of at least one asymptotically stable subsystem and is therefore "conservative" compared to the ones in the literature that accommodate families with all unstable systems [11,10]. On the other hand, we do not require the unstable systems in the family to form an asymptotically stable combination (e.g., in [11,10]).