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Finite abstractions with robustness margins for temporal logic-based control synthesis ☆

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ABSTRACT

This paper introduces a notion of finite abstractions that can be used to synthesize robust controllers for dynamical systems from temporal logic specifications. These finite abstractions, equipped with certain robustness margins, provide a unified approach to various issues commonly encountered in implementing control systems, such as inter-sample behaviors of a sampled-data system, effects of imperfect state measurements and unmodeled dynamics. The main results of this paper demonstrate that the robustness margins can effectively account for the mismatches between a control system and its finite abstractions used for control synthesis. The quantitative nature of the robustness margins also makes it possible to study the trade-offs between the performance of controllers and their robustness against various types of adversaries (e.g., delays, measurement errors, or modeling uncertainties). We use a simple adaptive cruise control (ACC) example to illustrate such robustness–performance trade-offs.

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1. Introduction

Designing hybrid controllers from high-level specifications using abstraction-based, hierarchical approaches has gained increased popularity over the last few years (see, e.g., [1–11]). The typical workflow of these approaches is as follows: (i) computation of finite abstractions of the system to be controlled, (ii) discrete synthesis based on the computed abstractions and desired specifications to obtain a discrete strategy, (iii) hybrid implementation of the discrete control strategy to ensure correctness of the overall system. In particular, how to compute finite abstractions of nonlinear control systems has received special attention (see [12,13] and references therein), as it is the first and most important step in ensuring the overall correctness of such approaches.

One advantage of using abstraction-based methods is that they can provide a feedback solution, as opposed to open-loop trajectory generation strategies [14,15]. While feedback has the potential to reduce the effects of disturbances and deal with sensing and modeling uncertainties, it remains unclear how to establish robustness of a hybrid feedback controller obtained from abstraction-based methods when the requirements are given in a high-level temporal logic. Motivated by this question, in this paper, we present a unified notion of finite abstractions that can be used to synthesize robust hybrid controllers from high-level specifications. These finite abstractions are equipped with additional robustness margins to

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account for imperfections in measurements and/or models. More specifically, by focusing on temporal logic specifications and nonlinear control systems, we show that, when the abstractions comply with these margins with respect to a nominal dynamical system, then it is possible to synthesize a hybrid control strategy that remains valid for a family of perturbed dynamical systems (i.e., that can be represented as the nominal dynamical system subject to uncertainty).

The main results of this paper show that it is possible to establish robustness against various issues that are commonly encountered in implementing control systems, namely inter-sample behaviors of a sampled-data system, effects of imperfect state measurements, unmodeled dynamics, jitter and delays within an abstraction-based framework. Such issues have been studied extensively for stability analysis, but they received less attention in the context of temporal logic-based control for dynamical systems. The quantitative nature of the robustness margins also provides explicit trade-offs between the performance of the hybrid controllers being designed and their robustness against various types of adversaries (e.g., delays, measurement errors, or modeling uncertainties).

A preliminary version of this paper appeared in [16]. The current paper differs from [16] in a number of ways. First, we provide a more in-depth discussion of abstractions and robustness margins both for continuous-time and for discrete-time systems. Second, in this paper, we define the robustness margins to be vector-valued parameters. This allows the use of hyper-boxes for computing abstractions and makes the abstractions less conservative as demonstrated in the examples section. Third, we discuss in this paper the trade-offs between robustness and performance of synthesized controllers using a new example on adaptive cruise control design. Fourth, we have added a more detailed explanation of the implementations of discrete control strategies, including a formal definition of continuous implementations of discrete strategies and new block diagrams showing the details of digital implementations. Making the implementation semantics explicit is crucial to talk about the correctness of the closed-loop system. Fifth, we have added a detailed model description for the case with imperfect state measurements. Finally, we have expanded the related work section and added several new remarks to discuss more about relevant work.

The rest of the paper is organized as follows. Preliminaries on temporal logics and control system models are given in Section 2. Finite abstractions with robustness margins are introduced in Section 3. The main results that demonstrate the effectiveness of the new abstraction framework are presented in Section 4. Section 5 discuses abstractions with robustness margins for discrete-time control systems. An example on vehicular cruise control is used to illustrate the results in Section 6, highlighting robust-performance trade-offs. Some related work is discussed in Section 7.

2. Preliminaries

Notation. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; given an n-vector $x = (x_1, \ldots, x_n)$ in $\mathbb{R}^n$, let $|x| = (|x_1|, \ldots, |x_n|)$, i.e., the n-vector obtained by taking entry-wise absolute value of x; given two n-vectors $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$, $x \le y$ means $x_i \le y_i$ for all $i \in \{1, \ldots, n\}$ ($x < y$, $x > y$, and $x \ge y$ are similarly defined); an n-vector x is said to be positive if $x > 0 \in \mathbb{R}^n$; given n-vectors $\delta \ge 0$ and x, let $B_\delta(x) := \left\{x' \in \mathbb{R}^n : \left|x' - x\right| \le \delta\right\}$; let $\mathbb{R}^+$ denote the nonnegative real line; given an interval $I \subseteq \mathbb{R}^+$ and $U \subseteq \mathbb{R}^m$, U^I denotes the set of control input signals from I to U; given a function f, $\mathrm{dom}(f)$ denotes its domain; given a scalar $r > 0$, $\mathcal{C}_r$ denotes the space of $\mathbb{R}^n$-valued continuous functions on $[-r, 0]$.

2.1. Linear temporal logics

We use the stutter-invariant fragment of linear temporal logic (denoted by $\mathrm{LTL}_{\setminus\bigcirc}$ [17], which means LTL without the next operator $\bigcirc$) to specify system properties. This logic consists of propositional logic operators (e.g., **true**, **false**, *negation* ($\neg$), *disjunction* ($\vee$), *conjunction* ($\wedge$) and *implication* ($\rightarrow$)), and temporal operators (e.g., *always* ($\Box$), *eventually* ($\diamond$), *until* ($\mathcal{U}$) and *release* ($\mathcal{R}$)).

The syntax of $\mathrm{LTL}_{\setminus\bigcirc}$ over a set of atomic propositions Π is defined inductively follows:

- **true** and **false** are $\mathrm{LTL}_{\setminus\bigcirc}$ formulas;
- an atomic proposition $\pi \in \Pi$ is an $\mathrm{LTL}_{\setminus\bigcirc}$ formula;
- if φ and ψ are $\mathrm{LTL}_{\setminus\bigcirc}$ formulas, then $\neg\varphi$, $\varphi \vee \varphi$, and $\varphi \mathcal{U} \varphi$ are $\mathrm{LTL}_{\setminus\bigcirc}$ formulas,

where atomic propositions are statements on a certain state space X. A labeling function $L : X \to 2^\Pi$ maps a state to a set of propositions that hold true for this state.

Negation normal form (NNF): All $\mathrm{LTL}_{\setminus\bigcirc}$ formulas can be transformed into negation normal form [18, p. 132], where

- all negations appear only in front of the atomic propositions[1];
- only the logical operators **true**, **false**, $\wedge$, and $\vee$ can appear; and

[1] Hence all negations can be effectively removed by introducing new atomic propositions corresponding to the negations of current ones. We assume this has been done for all $\mathrm{LTL}_{\setminus\bigcirc}$ formulas involved in this paper. This is for technical convenience in proving the main results of the paper, where we can change the labels of states with atomic propositions consistently, rather than having to deal with atomic propositions under negations separately.

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