Nonlinear Analysis: Hybrid Systems 22 (2016) 31–42

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Stochastic stability analysis and L_∞-gain controller design for positive Markov jump systems with time-varying delays

Shuo Li, Zhengrong Xiang *

School of Automation, Nanjing University of Science and Technology, Nanjing, 210094, People's Republic of China

ARTICLE INFO

Article history:
Received 17 October 2014
Accepted 28 February 2016

Keywords:
Positive systems
Markov jump systems
Stochastic stability
L_∞-gain controller design
Time-varying delay
Linear programming

ABSTRACT

This paper is concerned with stochastic stability analysis and L_∞-gain (also called L_∞-induced norm) controller design for positive Markov jump systems (MJSs) in continuous-time context by means of a linear programming (LP) technique. Firstly, by introducing an equivalent deterministic positive continuous-time linear system, we propose necessary and sufficient conditions of stochastic stability with a prescribed L_∞-gain performance in LP form for the underlying system. Then based on the obtained results, an effective method for designing a desired L_∞-gain controller is established. All the proposed conditions are solvable in terms of LP with additional parameters. Finally, a numerical example is given to illustrate the effectiveness of the present method.

1. Introduction

In practice, physical systems involve variables that have nonnegative property, like the population of human, plants or animals, concentration of substances, absolute temperature [1]. Such systems are referred to as positive systems, whose states and outputs are nonnegative whenever the initial conditions and inputs are nonnegative [2,3]. Many applications of positive systems can be found in practice, including engineering, economics, social science, ecology, biomedicine, communications [4–6]. For positive systems, due to the special positivity constraint, it is sometimes more appropriate to adopt L_∞-gain or L_1-gain rather than L_2-gain as the system performance measure [7–9]. In addition, ∞-norm provides a useful description for positive systems because it accounts for the maximal value of the quantities. As in compartment systems, one may be interested in analyzing the maximal mass in a single compartment with certain amount of input mass. For positive systems, one often applies the co-positive linear Lyapunov function rather than the traditional quadratic Lyapunov function as a valid candidate to discuss the control synthesis because the former will yield less conservative stability conditions [10], and the corresponding existence conditions are formulated in the Linear programming (LP) form rather than the Linear matrix inequality (LMI) form. Although both the LP problem and the LMI problem can be solved by efficient algorithms, the LP technique is simpler and possesses a numerical advantages than the LMI approach [11]. Thus it is more efficient to adopt LP approach to deal with the control synthesis of positive systems [7,12–16].

On the other hand, it is widely known that the reaction of real-world systems to exogenous signals is always not instantaneous and is affected by certain time delays, such as transportation systems and networked control systems [17]. Time delay frequently leads to instability and undesirable system performance. Recently, studies of positive time-delayed systems have attracted more and more attention, such as stability and stabilization analysis [6,18], fault detection [19], and finite-time control [20,21].

* Corresponding author.
E-mail addresses: lishuojx@126.com (S. Li), xiangzr@mail.njust.edu.cn (Z. Xiang).

http://dx.doi.org/10.1016/j.nahs.2016.02.004

Furthermore, dynamic systems subject to random abrupt variations can be modeled as Markov jump systems (MJSs), such as networked control systems [22], manufacturing systems [23], and fault-detection systems [24]. Positive Markov jump systems, as a special class of MJSs, provide a unified framework of mathematical modeling for many dynamic systems, like network employing TCP in communication systems, competitive electricity markets faced with uncertainty variations, the analysis of electricity price time series [25–28]. For a positive MJS, it is an area that is only beginning to be investigated, and there is further room for investigation. It should be pointed out that the stabilization of positive MJSs would be full of theoretical challenge, since one needs to consider not only the stability of subsystems as well as the closed-loop system governed by a Markov chain, but also the constrained positivity of the system.

Recently, there have been a limited amount of results on positive MJSs [11,29,30]. To mention a few, [29] deals with the stability analysis, as well as synthesis design for positive linear systems subject to random Markovian switching. Sufficient conditions for stochastic stability of continuous-time or discrete-time positive MJSs [11] were proposed in LP form. Necessary and sufficient conditions are given in [30] such that the discrete-time positive MJS is stochastically stable with an L_1-gain performance, which is the first paper to consider discrete-time positive MJSs with exogenous disturbance. It is widely recognized that many previous results of positive MJSs only consider the case that the transition probability is completely known [11,29], which may lead to some conservativeness. Moreover, to the best of our knowledge, most of the aforementioned results on positive MJSs [11,29,30] are based on nominal system without taking time delay and exogenous disturbance into account. As time delay and exogenous disturbance are frequently encountered in practice, it is necessary and significant to further consider positive MJSs with those two kinds of phenomenons. When taking time delays and exogenous disturbance into account, the problem of stability analysis and control synthesis becomes more complicated and challenging. However, until now, no relevant work on this kind of system has been published, which motivates our investigation.

In this paper, we will consider the stochastic stability analysis and L_∞-gain controller design for positive MJSs with time-varying delays. The main contributions of this paper include: (1) Based on the stability result for the deterministic positive linear systems with time-varying delays, a necessary and sufficient condition of stochastic stability is obtained for the first time with regard to positive continuous-time MJSs with time-varying delays; (2) The exact computation of L_∞-gain index of a stochastically stable positive continuous-time MJS is firstly presented by introducing an corresponding "equivalent" deterministic positive continuous-time linear system, and a necessary and sufficient condition which guarantees that the continuous-time MJS is stochastically internally stable as well as satisfies the prescribed performance index is given in LP form. (3) By considering uncertainties in the system matrices and transition probabilities, some robust L_∞-gain performance analysis results are obtained for the underlying system; (4) A sufficient condition on the existence of a desired L_∞-gain state feedback controller is presented based on the previously obtained results;

The rest of this paper is organized as follows. Some preliminaries are given in Section 2. Section 3 analyzes the stochastic stability, L_∞-gain performance, and robust L_∞-gain performance for positive continuous-time MJSs with time-varying delays. The L_∞-gain state feedback controller design problem is solved in Section 4. A numerical example is presented to show the effectiveness and applicability of the theoretical results in Section 5. Section 6 concludes the paper.

Notations: Z and Z_+ are the sets of nonnegative and positive integers, respectively. $A \succeq 0(\preceq, \succ, \prec)$ means that all entries of matrix A are nonnegative (nonpositive, positive, negative); $A \succ B(A \succeq B)$ means that $A - B \succ 0(A - B \succeq 0)$. A^T means the transpose of matrix A. $R(R_+)$ is the set of all real (positive real) numbers; $R^n(R^n_+)$ is n-dimensional real (positive real) vector space; $R^{m\times n}$ is the set of all $m \times n$-dimensional real matrices. I_n denotes the n-dimensional identity matrix and 1_n means the all-ones vector in R^n. $[A]_{ij}$ means the (i, j)-th entry of a matrix A and $[x]_i$ means the ith entry of a vector x. $diag(A_i)$ denotes the matrix formed with A_i in the diagonal and zero else, $i = 1, 2, \dots, N$. $E\{\cdot\}$ means the mathematical expectation of $\{\cdot\}$. $\|\cdot\|_2$ represents the Euclidean norm and is denoted by $\|x\|_2 = (\sum_{i=1}^n [x]_i^2)^{1/2}$, where $x \in R^n$. $\|x\|_\infty = \max_{1\le i\le n} |[x]_i|$ is the ∞-norm of a vector x in R^n, $\|A\|_\infty = \max_{1\le i\le m} \sum_{j=1}^n \left|[A]_{ij}\right|$ is the ∞-norm of a matrix A in $R^{m\times n}$. $\|\nu\|_{L_\infty} = \int_{t=0}^{\infty} E\{\|\nu(t)\|_\infty\}\, dt$ is the L_∞-norm of a stochastic signal $\nu(t)$, $t \ge 0$, L_∞ means the space of signal $\nu(t)$ having finite L_∞-norm, that is, $L_\infty = \left\{\nu(t), t \ge 0 | \, \|\nu\|_{L_\infty} < \infty\right\}$. $L_\infty^+ = \{\nu(t)|\nu(t) \in L_\infty, \nu(t) \succeq 0, \forall t \ge 0\}$. $\otimes$ denotes the Kronecker product. All matrices are assumed to have compatible dimensions, if their dimensions are not explicitly stated.

2. Problem formulation and preliminaries

Given a probability space (Ξ, Υ, Θ), where Ξ, Υ, and Θ represent, respectively, the sample space, the algebra of events, and the probability measure, the following continuous-time MJS with time-varying delay is considered:

$$\Sigma_{MC} : \begin{cases} \dot{x}(t) = A(r_t)x(t) + A_d(r_t)x(t - d(t)) + B(r_t)w(t), \\ z(t) = C(r_t)x(t) + C_d(r_t)x(t - \tau(t)) + D(r_t)w(t), \\ x(t_0 + \theta) = \varphi(\theta), \quad \theta = [-H, 0], \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state, $w(t) \in R^p$, $z(t) \in R^q$ are the disturbance input and controlled output, respectively, which belong to L_∞; the time delay are bounded, that is, $0 \le \underline{d} \le d(t) \le \overline{d}$ and $0 \le \underline{\tau} \le \tau(t) \le \overline{\tau}$; $H = \max\left\{\overline{d}, \overline{\tau}\right\}$, $\varphi(\theta)$ is the initial condition of the system; $t_0 = 0$ is the initial instant.

The $\{r_t, t \ge 0\}$ represents the jumping process and takes values in a finite set $S = \{1, 2, \dots, N\}, N \in Z_+$. For continuous-time systems, the jumping process is a continuous-time, discrete-state homogeneous Markov process and has the transition