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$\mathcal{H}_2$ and $\mathcal{H}_\infty$ control of time-varying delay switched linear systems with application to sampled-data control[☆]

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ABSTRACT

This paper deals with switched linear systems subject to time-varying delay. The main goal is to design state and output feedback switching strategies preserving closed-loop stability and a guaranteed $\mathcal{H}_2$ or $\mathcal{H}_\infty$ performance. The switching strategies are based on a generalization of a recent extended version of the small gain theorem and do not require any assumption on the continuity of the delay and its time-variation rate. The key point to obtain the design conditions is the adoption of an equivalent switched linear system where the time-varying delay is modeled as a norm-bounded perturbation. Moreover, with this approach, it is possible to deal with sampled-data control systems. All conditions are formulated in terms of Lyapunov–Metzler inequalities, which allow the maximization of an upper bound on the time-delay preserving stability and guaranteed performance. Numerical examples are discussed in order to illustrate the effectiveness of the design approach.

1. Introduction

The recent literature displays an increasing interest in the study of switched systems subject to time-delays, which can be used to represent many classes of real-world situations including measurement and actuators delays [1], information transmission delays [2,3], neural networks [4], among others. Relevant books in the area of time-delay systems and switched systems are [5,6], among others.

In the control community, consistent attention has been devoted to the special class of switched linear systems with time-delays, see for instance [7–9]. When the feedback stabilization is considered, most of the contributions proposed so far are related to the case where the delay is constant. Among the recent papers, [10] proposed a stabilizing switching rule based on a Riccati-type common Lyapunov functional approach and assuming a condition on the time-delay. In [11], both delay-independent and delay-dependent strategies for the state-feedback $\mathcal{H}_\infty$ control of switched linear systems were worked out, relying on suitable Lyapunov–Krasovskii functionals. Further advances were achieved in [12], where output-feedback delay-independent switching laws are proposed, and [13], which provides a new perspective based on an extended small-gain theorem. On the other side, new research efforts are being made to deal with time-varying delays. In [14], exponential

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stability and $\mathcal{L}_2$-gain were studied, considering switching signals with average dwell time. Ref. [15] addressed stability analysis of systems with uncertain time-varying delays. In [16], a time-varying delay acting on both the state and the control was considered for the nonlinear case by an approach based on Lyapunov–Krasovskii functionals.

Differently from [17,18], where the switching is considered as a given exogenous perturbation characterized by presenting some dwell time and/or average dwell time, switched linear systems subject to time-varying delays, for which the switching is a control variable, are the main focus of this paper. It is devoted to the use of switching for stabilization and $\mathcal{H}_2$ or $\mathcal{H}_\infty$ guaranteed performance optimization by either state and output feedback, under the assumption that the time-varying delay evolves within specified bounds. Our results are based on the concept of Lyapunov–Metzler inequalities, originally proposed in [19,20], as a tool to derive stabilizing switching strategies. Beyond stabilization, controlled switching has been exploited to improve performance when compared to the non-switched situation, see [20–22], or to enhance robustness of the feedback control system [13]. In particular, the present contribution stands as a generalization of Ref. [13], which has provided delay-dependent stability conditions for the design of a stabilizing switching rule, but only when the delay is time-invariant and without considering any performance index. Furthermore, here, we extend the results of [23] by including the determination of switching strategies ensuring prescribed $\mathcal{H}_2$ or $\mathcal{H}_\infty$ guaranteed performances which, to the best of the author's knowledge, is a problem not treated in the literature to date.

In order to formulate our results, we resort to the extended formulation of the small-gain theorem originally provided in [13]. In this way, the original switched linear system is reformulated as a feedback interconnection between a delay-free switched linear subsystem and a norm-bounded perturbation block. Then, the stability of the overall system is imposed by designing the switching law so that the $\mathcal{L}_2$-induced norm of the delay-free subsystem is less than a prescribed value. Finally, we specify $\mathcal{H}_2$ and $\mathcal{H}_\infty$ performance requirements accordingly.

An interesting feature about the problem considered in this paper is its possible application to control design of sampled-data systems. In fact, it is well known that sampling can be represented in terms of delays, see for instance [24,25] and the references therein. In particular, nonuniform sampling can be equivalently represented as a time-varying delay acting on the input variables of the system [26]. The recent results on state feedback sampled-data control design for linear time invariant systems reported in [25] are used to certificate the ones proposed in this paper.

The paper is organized as follows: in Section 2 we formulate the control problem and review relevant preliminary results, specially related to the use of the small gain theorem for the class of problems under consideration. Section 3 introduces a way to rewrite the system equations enabling its study from the mentioned perspective. Performance optimization is the subject of Section 4, where we introduce both state and output feedback control synthesis results. Section 5 discusses the application of our techniques to sampled-data control systems design with the support of a numerical example. Finally, there are some concluding remarks.

The notation is standard. The identity matrix of any dimension is denoted by I. For real matrices or vectors, the symbol $(')$ indicates transpose. For any square matrix $\mathrm{Tr}(\cdot)$ represents its trace. For a symmetric matrix, the symbol $(\bullet)$ denotes each of its symmetric blocks and $Q > 0$ $(Q < 0)$ indicates that the symmetric, real matrix Q is positive definite (negative definite). The set of natural numbers is $\mathbb{N}$ and $\mathbb{K} = \{1, 2, \ldots, N\}$. The squared norm of a signal $\xi(t)$ defined for all $t \geq 0$, denoted by $\|\xi\|_2^2$, is equal to $\int_0^\infty \xi(t)'\xi(t)dt$. The set of all signals such that $\|\xi\|_2^2 < \infty$ is denoted by $\mathcal{L}_2$. For a real matrix M, the Hermitian operator $\mathrm{H_e}\{\cdot\}$ is defined as $\mathrm{H_e}\{M\} = M + M'$. The set $\mathcal{M}$ is composed by all Metzler matrices $\Pi = \{\pi_{ji}\} \in \mathbb{R}^{N\times N}$, with non-negative off-diagonal elements satisfying the constraints $\sum_{j\in\mathbb{K}} \pi_{ji} = 0, \ \forall i \in \mathbb{K}$. Given a continuous (not necessarily differentiable) function $f(t)$, the Dini derivative is defined as $D^+f(t) = \limsup_{\Delta t\to 0^+} (f(t + \Delta t) - f(t))/\Delta t$. Finally, the symbol "$\circ$" indicates the application of a linear input–output operator to a signal.

2. Problem formulation and preliminaries

Consider a switched linear system described by

$$\dot{x}(t) = A_\sigma x(t) + A_{d\sigma} x(t - h(t)) + H_\sigma w(t) \tag{1}$$

$$z(t) = E_\sigma x(t) + E_{d\sigma} x(t - h(t)) + G_\sigma w(t) \tag{2}$$

$$y(t) = C_\sigma x(t) + C_{d\sigma} x(t - h(t)) + D_\sigma w(t) \tag{3}$$

where $h(t)$ is the time-varying delay and the vectors $x \in \mathbb{R}^{n_x}$, $w \in \mathbb{R}^{n_w}$, $y \in \mathbb{R}^{n_y}$ and $z \in \mathbb{R}^{n_z}$ are the state, the external input, the measured output and the controlled output, respectively. It is supposed that $h(t)$ satisfies the constraint $0 \leq h(t) \leq h_m$ for all $t \geq 0$ and the system evolves from zero initial condition, that is $x(t) = 0$, $-h_m \leq t \leq 0$. No assumption on continuity and on the time derivative $\dot{h}(t)$ is required. We only suppose that $h(t)$ is piecewise continuous. The switching function, denoted by $\sigma(\cdot)$, is the unique control variable to be determined. It selects at each instant of time $t \geq 0$ a subsystem $\mathcal{P}_i$ among the set $\{\mathcal{P}_1, \ldots, \mathcal{P}_N\}$ of available ones defined by matrices

$$\mathcal{P}_i := \begin{bmatrix} A_i & A_{di} & H_i \\ E_i & E_{di} & G_i \\ C_i & C_{di} & D_i \end{bmatrix} \tag{4}$$

of compatible block dimensions.