Nonlinear Analysis: Hybrid Systems 22 (2016) 72–97

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Individuals, populations and fluid approximations: A Petri net based perspective☆

Manuel Silva *

Instituto de Investigación en Ingeniería de Aragón (I3A), Universidad de Zaragoza, María de Luna 1, 50018, Spain

ARTICLE INFO

Article history:
Received 12 June 2015
Accepted 23 March 2016

Keywords:
Discrete event system
Petri net
Fluidization
Piecewise affine system
Symmetries
Decolorization

ABSTRACT

Discrete Event Systems (DES) theory and engineering are mainly driven by needs that arise in many different human-made systems (manufacturing, communications, logistics, workflow management, traffic, etc.). With the accelerated increase in the complexity and size of new technological constructions, the *state explosion problem* in DES analysis and synthesis becomes more and more acute.

Two traditional conceptual and complementary ways of dealing with the computational complexities in the Petri nets (PN) framework are *structure theory* (that investigates the relationship between the behavior of a net system and its structure) and *fluid relaxations*, here leading to particular classes of hybrid systems. In the second case, the expected computational gains for analysis and synthesis problems are usually achieved at the expense of the fidelity or accuracy of the relaxed model. This invited overview will mainly focus on the second strategy, nevertheless always interspersed with basic structural concepts and methods. Using an example-driven approach, starting with a DES "view of the system", the legitimization and improvement of fluidization process, the aggregation of local states by symmetries and the decolorization of models will be briefly addressed, together with reflections about the analysis of the new models obtained.

As the linearization of a continuous dynamical system, the *fluidization* of a DES is a relaxation that has to be used with care, depending on the problem at hand. This abstraction is here considered from two complementary perspectives: at *logical* and at *performance* levels, both for untimed and timed PNs. On the one hand, the expressive power of timed fluid PNs under *infinite server semantics* is such that the simulation of *Turing machines* is possible. From a complementary perspective, the expression of modeling capabilities such as *non-monotonicities* and *bifurcations* may also be revealed for steady-state behaviors. *Symmetries* (more generally, *lumping*) seek to group together "equivalent" behaviors and *decolorization* seeks to abstract identities, in order to create new collectivities of processes and resources. The *synergy* between symmetry-decolorization state-aggregation approaches and fluid relaxations is highlighted. In fact, the first approaches not only reduce the state space, but also "produce" populations, thus proceed upgrading the applicability of fluidization. Opening the window, related issues such as control, optimization, observation or diagnosis are briefly pointed out. For conciseness, this work is limited to fully fluid (or continuous) PN models and their relationships with the corresponding discrete systems.

☆ This work has been partially supported by the Aragonese Government and the European Social Fund under grant ref. T27- GISED research group, and by CICYT - FEDER grant DPI2014-57252-R.

* Tel.: +34 976761952; fax: +34 976761914.
E-mail address: silva@unizar.es.

http://dx.doi.org/10.1016/j.nahs.2016.03.005
1751-570X/

1. Introduction

The analysis and synthesis of *Discrete Event* "views" of dynamic *Systems* (DES), suffer from the *state explosion problem*. In this work Petri Nets are used as the modeling framework for DES. They constitute a broad family of related formalisms especially suited to dealing with parallel and distributed evolutions, and whose behavior are characterized by synchronization and sharing phenomena. All these formalisms enjoy some basic relevant features, such as intuitive graphical representations, and *locality* of the states and the state-changes. Among the consequences, "true concurrency" (basically, a non-interleaved semantics) can be modeled, what results in *temporal realism* [1]. In untimed net models, the state variables derive from the *places* (represented by circles), while their values are called *markings* (usually visualized by dots inside the places); the global state of a system is *numerically* quantified, usually expressed in a vector form; state evolutions are caused by events and are depicted by objects named *transitions* (symbolized with rectangles or bars). The underlying logic in the dynamic behavior of a Petri net system is of the *consumption/production* type, thus it is non-monotonous.

In the sequel it is assumed that the reader is aware of the most elementary concepts of Petri Nets, basically at the level of *Place/Transition nets* (P/T-nets), by default simply called *Petri Nets* (PNs). For gentle introductions to the field, complementary basic surveys are [2], [3], and [4]. The latter also discusses some fully *fluid* or *continuous* models (terms here considered to be synonymous) and some *partially fluid* or *hybrid* models. Topics related to the Petri nets modeling paradigm are studied in well over a hundred thousand papers and reports. A recent broad historical perspective on the field is provided in [5].

Continuous "relaxations" of DES are not new. They are derived as fluid limit cases of discrete "views" of systems (models) when the populations become very large. In fluid PN models, the firing amount of all transitions (thus the marking of all places) are relaxed to non-negative real quantities. Fluid representations are usually inferred from the "macroscopic" averaging of enormous amounts of "microscopic" discrete states and events. In this context, fluidization of *Queuing Networks* (QNs) already received attention more than four decades ago [6], and there is abundant literature on the topic. The introduction of fluidization (or continuization) in the Petri net paradigm dates back to 1987 [7]. R. David explicitly states (see [8, p. IX]) that the source of inspiration was the fluidization of models for the performance evaluation of production lines (manufacturing domain). It is simply a coincidence that, at the same meeting in Zaragoza, working with the *fundamental* or *state-transition equation* of the net system, was proposed the systematic use of linear programming techniques for the structural analysis of Petri nets (see [9] for a more accessible version). In fact, this second approach can initially be "rephrased" as relaxing *Integer Programming* into *Linear Programming* in order to obtain: necessary or sufficient conditions for *qualitative* properties (such as boundedness, deadlock-freeness, etc.); or bounds for *quantitative* ones (for example, of the maximum number of tokens in a place in an untimed model; or of the throughput of a transition – the number of its firings per time unit – in a timed model [10]).

With a certain flavor in systems engineering and automatic control, broad perspectives on continuous PNs are provided in chapters 4 and 5 of book [8], in survey [11], and in Chapters 18, 19 and 20 of book [12]. Starting in 2005, the fluidization of *Process Algebras* (PAs) has recently received a significant attention (for a global perspective, see [13]). With regard to some relationships of fluid PNs and fluid PAs, see [14,15].

Several formalisms deal directly with fluid "views" of systems that occasionally can be "more naturally perceived" as discrete (for example, predator–prey systems). Like Petri nets, *Forrester Diagrams* (FDs) [16] are bipartite and have their roots in the very beginning of the 1960s. They are also expressively called *Stock and Flow Diagrams* (for their connections with time continuous PNs, see [17,18]). *Stochastic Flow Models* (SFMs), provide alternative timed continuous "views" of DES, where their use is recognized "for control and optimization of communication networks in which detailed discrete event models become impractical" [19].

State-aggregation techniques (symmetry reduction, lumping or decolorization) will be very briefly mentioned as examples of complementary abstractions that can be performed. Some of these allow the evolution from models with *individuals* to others with *populations* (i.e., to more compact formal representations in which emphasis is placed on the collective behavior). Population based modeling is especially useful for representing the dynamics of large-scale distributed systems, most frequently consisting of a great number of independent-equally behaved components.

Once models with populations are available, fluidization can be performed and the legitimization of continuous approximations such as *Ordinary Differential Equations* (ODEs) or *Stochastic Differential Equations* (SDEs) can be established. The important point is that with this abstraction (or relaxation) the larger the population, the better the approximation usually is, while the computational costs may decrease exponentially. Obviously, the relationship between the untimed and timed properties of the DES model and the corresponding characteristics of their continuous approximations is a very important issue, unfortunately not frequently addressed.[1] In addition, the question of the expressive power of the formalisms obtained will be briefly mentioned.

Introducing new thoughts, this essay is a broad overview of facts, possibilities and problems. Simple examples will help to drive the task forward. In Section 2, two examples are used to present the easy to accept notion that with growing populations the relative error due to fluidization diminishes in many practical cases. Fluid untimed (autonomous or non-deterministic) and timed PN models are the subjects of Sections 3 and 4, respectively.

[1] Forrester Diagrams and Stochastic Flow Models are timed and continuous formalisms. With PNs the relationships of discrete models and their continuous relaxations can be analyzed. Moreover, PN systems may be untimed (i.e., fully non-deterministic) and timed.